Graphics 2007/2008

Midterm Exam

Thu, Dec 13, 2007, 13:15–15:00

- **Do not open this exam until instructed to do so. Read the instructions on this page first.**
- You may write your answers in English, Dutch, or German.
- You may not use books, notes, or any electronic equipment (including your cellphone, even if you just want to use it as a clock).
- Please put your student ID on the table so we can walk around and check it during the exam.
- Write down your name and studentnumber on every paper you want to turn in. Additional paper is provided by us. You are not allowed to use your own paper.
- You have 1 hour and 45 minutes for this exam. If you finish early, you may hand in your work and leave, except for the first half hour of the exam.
- The test consists of 3 problems related to vectors, matrices, and transformations, respectively. The maximum number of points you can score is 18 (6 for each problem). You need 16 points to get the best possible grade.

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Problem 1: Vectors, basic geometric shapes, intersections

Subproblem 1.1 [2 pt] Which of the following answers are correct? Write down *all* possible solutions (i.e. there might be more than one correct answer to each question!). It is *not* needed to give an explanation.

The intersection of …

(i)	… a line and a sphere can have exactly	[a] 0	[b] 1	[c] 2	or	[d] ∞	solutions
(ii)	… a line and a plane can have exactly	[a] 0	[b] 1	[c] 2	or	[d] ∞	solutions
(iii)	… a plane and a plane can have exactly	[a] 0	[b] 1	[c] 2	or	[d] ∞	solutions
(iv)	… three planes can have exactly	[a] 0	[b] 1	[c] 2	or	[d] ∞	solutions

Subproblem 1.2 [4 pt] Assume the following three points in $\mathbb{R}^3$:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$

(1.2a) Give a parametric equation of the line l through the points **a** and **b**.

(1.2b) What is the geometric interpretation of the parametric equation given in (1.2a)?

(1.2c) Calculate a normal vector **n** for the plane defined by **a**, **b**, and **c**

(1.2d) Create an implicit representation of the plane defined by **a**, **b**, and **c**.
(Note: ”Create” means that it is not sufficient to just write down the equation of the plane but that we should be able to recognize how you got this solution.)

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Problem 2: Matrices

Subproblem 2.1 [1 pt] Prove that matrix multiplication is not commutative, i.e. that in general $\mathbf{AB} \neq \mathbf{BA}$.

Subproblem 2.2 [3 pt] Assume the following three planes in $\mathbb{R}^3$:

$$\begin{array}{rcl} x + 2y + 8z & = & 11 \\ x + 4y + 12z & = & 17 \\ 4y + 10z & = & 14 \end{array}$$

(2.2a) Construct all intersection points of the three planes using Gaussian elimination.

(2.2b) What is the geometric interpretation of your solution?

Subproblem 2.3 [2 pt] Show that for a nxn-matrix $\mathbf{A}$, $\mathbf{AA}^T$ is a symmetric matrix
(i.e. you have to show that $c_{ij} = c_{ji}$ for any coefficient c_{ij} of the matrix $\mathbf{AA}^T$).

Problem 3: Transformations

Subproblem 3.1 [2 pt] Assume the following transformation matrices:

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Associate each of these matrices with one of the following statements: (Note: You only have to give the letter of the correct matrix **A**, **B**, **C**, or **D** for each statement, i.e. an explanaition is *not* needed.)

This transformation matrix represents ...
(i) ... a reflection on $y = 0$
(ii) ... a reflection on $x = 0$
(iii) ... a reflection on $x = y$
(iv) ... a point reflection in the origin

Subproblem 3.2 [1 pt] Write down the 3x3 matrix for a rotation by an angle of θ around the x-axis in $\mathbb{R}^3$.

Subproblem 3.3 [1 pt] Describe in your own words what happens to a vector $\mathbf{v}$ if you apply the following transformation matrix to it:

$$\begin{pmatrix} 2 & 0 & 0 & x_m \\ 0 & 2 & 0 & y_m \\ 0 & 0 & 2 & z_m \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Subproblem 3.4 [2 pt] The following matrix defines scaling (in $\mathbb{R}^2$) by a factor of a and b in x- and y-direction, respectively:

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

Prove that matrix multiplication with this scaling matrix is a linear transformation.