Graphics 2010/2011

T2

Final exam

Tue, July 5, 2011

Do not open this exam until instructed to do so.

Read the instructions on this page carefully.

- You may write your answers in English or Dutch.
  Use a pen, not a pencil. Avoid usage of the color red.

- Write down your name and student number on every paper you want to turn in.
  Additional paper is provided by us. You are not allowed to use your own paper.

- You may **not** use books, notes, or any electronic equipment
  (including your cellphone, even if you just want to use it as a clock).

- You have **max. 2 hours** to work on the questions.
  If you finish early, you may hand in your work and leave,
  except for the first half hour of the exam.

- When you hand in your work, have your student ID ready for inspection
  and write your name and ID on the list of participants.

- The exam consists of 10 problems printed on 7 pages (including this one).
  It is your responsibility to check if you have a complete printout.
  If you have the impression that anything is missing, let us know.

Good luck!

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Problem 1: Texture mapping

■**Subproblem 1.1 [1 pts]:** Give a procedure that creates a stripe texture. The stripes should have alternating colors *color1* and *color2* along the $Z-$axis. The width of all stripes should be π.

■**Subproblem 1.2 [0.5 pts]:** Which of the following statements are correct? List the correct number(s). (This is a multiple choice question. No explanation is required. Listing incorrect answers might result in deduction of points.)

1. Bump mapping causes an apparent change of geometry.

2. Bump mapping produces correct silhouettes of objects.

3. Bump mapping modifies the points for which we do the shading.

4. Bump mapping modifies the normals at the points for which we do the shading.

Problem 2: Perspective projection

■**[2.5 pts]:** Applying the perspective transform matrix P to a point $(x, y, z, 1)$ gives us the point $(x_s, y_s, z_s, 1)$ as follows:

$$P \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z\frac{n+f}{n} - f \\ \frac{z}{n} \end{pmatrix} \xrightarrow{\text{homogenize}} \begin{pmatrix} \frac{nx}{z} \\ \frac{ny}{z} \\ n+f - \frac{fn}{z} \\ 1 \end{pmatrix} = \begin{pmatrix} x_s \\ y_s \\ z_s \\ 1 \end{pmatrix}$$

Prove that the following statements for the value $z_s = n + f - \frac{fn}{z}$ are correct:
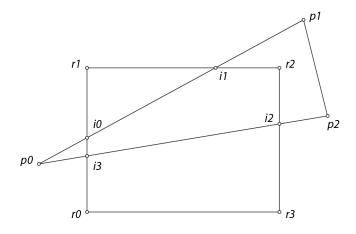
1. Values on the near plane stay on the near plane.

2. Values on the far plane stay on the far plane.

3. Values within the view frustum stay within the view frustum.

4. The order along the $Z-$axis for any two random values is preserved.

Problem 3: Clipping

■ **Subproblem 3.1 [0.5 pts]:** Draw an example for a case in which the Sutherland-Hodgman algorithm for clipping arbitrary polygons doesn't work correctly, i.e. the result is a degenerated polygon.

■ **Subproblem 3.2 [1 pts]:** Draw the graph that we get when we apply the Weiler-Atherton algorithm to clip the polygon defined by $p0, p1, p2$ on the clipping region defined by $r0, r1, r2, r3$.

Use the notation given in the image, always start with the lowest index number (i.e. $p0$ and $r0$), and don't forget to clearly mark what nodes in your graph represent incoming and outgoing intersections (e.g. by using a circle for incoming and a square for outgoing edges in your drawing).
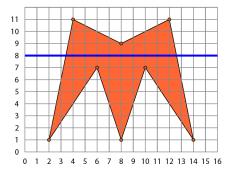

Problem 4: Backface culling

■ **[1.5 pts]:** Assume we have a triangle that is part of a closed polygon. The triangle defines a face $f(\vec{p}) = (3,2,0) \cdot \vec{p} - 3 = 0$ (notice that this is the implicit equation of a plane and $(3,2,0)$ is a normal vector of this plane).

1. Assume we place a camera at $\vec{e_1} = (2,1,0)$. Is it on the positive or negative side of $f(\vec{p})$? Prove or explain your answer.

2. Assume we the camera to $\vec{e_2} = (1,-4,0)$. On which side of $f(\vec{p})$ is it now – the positive or negative side? Prove or explain your answer.

3. Assume that the triangle is part of our view frustum in both cases. In which case do we remove it when we apply backface culling (none, e_1, e_2, or both)? Explain your answer.

Short explanations are sufficient but no credits will be given for correct answers with missing or wrong explanation!

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Problem 5: Triangle rasterization

■ **[2 pts]:** We want to apply the scanline conversion algorithm to rasterize the polygon shown in the image below.

1. The entries in the Edge Table specify the edges of our polygon. One of them is $1 : (2, 7, \frac{2}{3})$. Explain what each of the four numbers means (a short phrase for each number is enough to answer this quesion; no detailed explanation is needed).

2. The entries in the Active Edge Table specify the edges of our polygon that intersect with the current scanline. Give all entries of the scanline at position number 8.

Problem 6: Radiosity

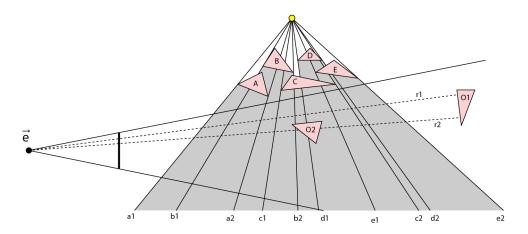■ **[1.5 pts]:** The radiosity B_i of a patch A_i can be calculated as follows:

$$B_i = E_i + \rho_i \sum_j B_j F_{ij}$$

1. The factor F_{ij} is called form factor from A_i to A_j and specifies the fraction of the energy leaving from patch A_i that arrives at patch A_j. Its value depends on three criteria. What are these?

2. What do the other three parameters describe that you find in this formula, i.e. what do E_i, ρ_i, and the B_js stand for?

Notice that you do not have to write down any formula here. A short verbal description is sufficient to get full credit (in most cases, just 1-2 words will be enough).

Problem 7: Shadows

■ **[1.5 pts]:** In the following image, you see a light source (the circle at the top) and five objects creating a shadow (the triangles labeled A till E). The faces of the shadow volumes created by these objects are labeled with $a1, a2, b1, b2, \ldots e1, e2$ for objects $A, B, \ldots E$ In addition, we have a camera position $\vec{e}$ and two objects $O1$ and $O2$.

We want to use the Stencil buffer to check if the two objects $O1$ and $O2$ are in the shadow or not. We are using a depth-pass approach.

1. Let's look at an intermediate step of the algorithm to fill the Stencil buffer. Assume that we have drawn the shadow faces $a1, a2, b1, b2, c1, d1$ and $e1$. At that time, what are the values in the entries of the Stencil buffer that correspond to the rays $r1$ and $r2$ illustrated by the dashed line in the image? (No explanation needed; just write down the corresponding values)

2. What are the values in these two entries after the algorithm is finished, i.e. once all shadow volume faces have been considered? (No explanation needed; just write down the corresponding values)

3. What does that mean for the two objects? Shortly explain your answer (a very short description is sufficient as long as it illustrates that you understood the algorithm and did not just look at the image to check if the objects are in the shadow or not).

Problem 8: Ray tracing: basics

■ **[0.5 pts]:** Which of the following statements are correct? List the correct number(s). (This is a multiple choice question. No explanation is required. Listing incorrect answers might result in deduction of points.)

1. Viewing rays for calculating orthographic views have the same direction.

2. Viewing rays for calculating orthographic views have the same origin.

3. Viewing rays for calculating perspective views have different origins.

4. Viewing rays for calculating perspective views have the same direction.

5. If the coefficients β and γ of the barycentric coordinates of an intersection point are both between 0 and 1, the point intersects with the triangle that defines this barycentric coordinates system.

6. If a closed planar polygon and a ray that we shoot from a point $\vec{p}$ into a random direction have an uneven number of intersections, then the point must be within the polygon.

Problem 9: Ray tracing: constructive solid geometry

■ **[1 pts]:** Assume we have two squares as illustrated in the image below. Using Constructive Solid Geometry (CSG), we want to create the gray object that is also illustrated in the image.
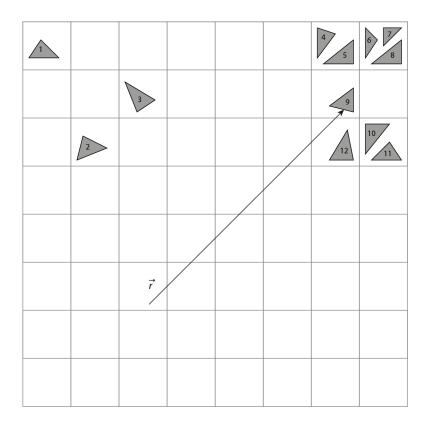

1. Let $S1$ denote all the points covered by the bigger square and $S2$ the ones covered by the smaller one. Write down the set operation that you have to do to describe all points covered by the gray object.

2. Now we want to calculate the intersection points of the ray defined by the vector $\vec{r}$ illustrated in the image with the newly generated gray object. Write down how we do this with CSG.

   Notice that no complex calculations are required here. You can read the values from the image. However, it is not sufficient to just write down the result. It should be clear that you know how to get it using CSG.

Problem 10: Faster ray tracing

■ **[1.5 pts]:** Assume we want to apply some space partitioning approach to the scene below in order to speed up our ray tracer.

1. We want to apply the Quadtree approach in order to particion the space in cells that contain at max. *three* objects. Starting with one huge cell that contains all triangles $[1,2,3,4,5,6,7,8,9,10,11,12]$ write down the cells that you get *in each step* of the algorithm.

   Write down the cells containing triangles using the same notation as for the starting cell indicated above. In addition, write down how many empty cells you have in each step.

2. If you do the intersection tests with the given ray $\vec{r}$ using the space partitioning that you just created with the Quadtree approach, how many false positives do you get?

3. How many false positives do you get if you apply Uniform Spacial Subdivision instead of the Quadtree approach when you use the same stop criteria (i.e. max. 3 objects per cell)?