Graphics (INFOGR 2011-2012): Midterm Exam (T1)

Friday, May 25, 2012, EDUC-GAMMA, 09:00-12:00 (duration: 2 hours)

StudentID / studentnummer	Last name / achternaam	First name / voornaam

Do not open the exam until instructed to do so!

Read the instructions on this page carefully!

- You may write your answers in English or Dutch. Use a pen, not a pencil. Do not use red or green.
- Fill in your name and student id at the top of this page, and write it on every additional paper you want to turn in.
- Answer the questions in the designated areas on these exam sheets. If you need more space, make a cross in the designated box at the end of the problem and continue writing on the additional paper provided by us. You are not allowed to use your own paper. On the additional paper, make sure to clearly indicate the problem number and start a new page for each problem.
- You may **not** use books, notes, or any electronic equipment (including your cellphone, even if you just want to use it as a clock).
- You have max. 2 hours to work on the questions. If you finish early, you may hand in your work and leave, except for the first half hour of the exam. When you hand in your work, have your **student ID** ready for inspection.
- The exam has 4 problems printed on 16 pages (including this one). It is your responsibility to check if you have a complete printout. If you have the impression that anything is missing, let us know.

Good luck / veel succes!

Do not write below this line

Problem 1 (Vectors)		sum (max. 20)
1.1 (max. 8)		
1.2 (max. 6)		
1.3 (max. 6)		

Problem 2 (Geometry)		sum (max. 30)
2.1 (max. 10)		
2.2 (max. 12)		
2.3 (max. 8)		

Problem 3 (Matrices)		sum (max. 25)
3.1 (max. 7)		
3.2 (max. 6)		
3.3 (max. 7)		
3.3 (max. 5)		

Problem 4 (Transf.)		sum (max. 25)
4.1 (max. 5)		
4.2 (max. 10)		
4.3 (max. 10)		

Total number of points: __________ Grade: __________

Problem 1: Vectors

■ Subproblem 1.1 [8 pts]: Multiple Choice Questions

1. The two vectors $(1,2)$ and $(-2,1)$ are …
   - A. …linearly dependent of each other.
   - B. …forming an orthonormal basis.
   - C. …perpendicular to each other.
   - D. …pointing in the opposite direction of each other.

This is a multiple choice question. Answer by marking the correct result.
An explanation is **not** required. In this case, there is only **one correct answer**.

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2. If s is a scalar value and $\vec{v}$, $\vec{w}$ are two vectors in 3D, then the result of $s + (\vec{v} \times \vec{w}) \cdot (\vec{w} \times \vec{v})$ is …
   - A. …a vector in 3D.
   - B. …undefined.
   - C. …a scalar.
   - D. …a $3x3$ matrix.

   Note: "$\times$" denotes the cross product, "$\cdot$" denotes the scalar product (also called dot product or inner product).

This is a multiple choice question. Answer by marking the correct result.
An explanation is **not** required. In this case, there is only **one correct answer**.

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3. Let θ be the angle between two vectors (neither of which is the null vector). If $90° < \theta < 270°$, the scalar product of these two vectors is …
   - A. …positive.
   - B. …negative.
   - C. …undefined.
   - D. …positive for $\theta < 180°$ and negative for $\theta > 180°$.

This is a multiple choice question. Answer by marking the correct result.
An explanation is **not** required. In this case, there is only **one correct answer**.

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4. If the scalar product of two unit vectors is zero, they are …
   - A. …linearly dependent.
   - B. …forming an orthonormal basis.
   - C. …at an angle of 45° or 225° to each other.
   - D. …pointing in the same direction.

This is a multiple choice question. Answer by marking the correct result.
An explanation is **not** required. In this case, there is only **one correct answer**.

■ **Subproblem 1.2 [6 pts]: Vector calculation**

Assume two vectors $\vec{u}$, $\vec{v}$ in 3D with $\vec{u} = (2, 2, 1)$ and $\vec{v} = (4, 4, 1)$.

1. Calculate the scalar product $\vec{u} \cdot \vec{v}$. (Note: the scalar product is also called dot product or inner product.)

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

2. Create a unit vector $\vec{u}'$ that points in the same direction as vector $\vec{u}$.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

3. Are the two vectors $\vec{u}$ and $\vec{v}$ linearly dependent or linearly independent? Shortly justify your answer.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

■ **Subproblem 1.3 [6 pts]: Vector characteristics**

Assume two vectors $\vec{w}$, $\vec{v}$ in 3D with $\vec{w} = \lambda\vec{v}$ and $\lambda = \frac{1}{2}$.
Prove that vector $\vec{w}$ is half as long as vector $\vec{v}$.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

Problem 2: Basic Geometric Entities

■ Subproblem 2.1 [10 pts]: Multiple Choice Questions

1. The equation $2x+y+z-10=0$ represents …
   - A. …the implicit representation of a line in 3D.
   - B. …the parametric representation of a line in 3D.
   - C. …the slope-intercept representation of a line in 3D.
   - D. …the implicit representation of a plane in 3D.
   - E. …the parametric representation of a plane in 3D.
   - F. …the slope-intercept representation of a plane in 3D.
   - G. none of the above

This is a multiple choice question. Answer by marking the correct result.
An explanation is **not** required. In this case, there is only **one correct answer**.

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2. Which of the following vectors are normal vectors of the vector (x,y) in 2D $(x,y \neq 0)$?
   - A. $(-x,y)$
   - B. $(x,-y)$
   - C. $(-x,-y)$
   - D. (y,x)
   - E. $(y,-x)$
   - F. $(-y,x)$
   - G. $(-y,-x)$
   - H. none of the above

This is a multiple choice question. Answer by marking the correct results.
An explanation is **not** required. In this case, **multiple answers might be correct**.
Marking wrong answers will slightly reduce your credit but only if your total credit for this question will not be negative.

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3. In a 3-dimensional space the equation $(x-2)^2+(y-2)^2+(z-2)^2-4=0$ represents …
   - A. …an implicit representation of a sphere with radius 2 and center $(2,2,2)$.
   - B. …an implicit representation of a sphere with radius 2 and center $(-2,-2,-2)$.
   - C. …an implicit representation of a sphere with radius 4 and center $(2,2,2)$.
   - D. …an implicit representation of a sphere with radius 4 and center $(-2,-2,-2)$.
   - E. …a parametric representation of a sphere with radius 2 and center $(2,2,2)$.
   - F. …a parametric representation of a sphere with radius 2 and center $(-2,-2,-2)$.
   - G. …a parametric representation of a sphere with radius 4 and center $(2,2,2)$.
   - H. …a parametric representation of a sphere with radius 4 and center $(-2,-2,-2)$.
   - I. none of the above

This is a multiple choice question. Answer by marking the correct result.
An explanation is **not** required. In this case, there is only **one correct answer**.

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4. The intersection of three planes in 3D can be ...?
   A. ...empty.
   B. ...a point.
   C. ...a line.
   D. ...a plane.
   E. ...a parallelogram.
   F. ...three lines that are parallel to each other.
   G. none of the above

This is a multiple choice question. Answer by marking the correct results.
An explanation is **not** required. In this case, **multiple answers might be correct**.
Marking wrong answers will slightly reduce your credit but only if your total credit for this question will not be negative.

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5. The following equation represents a plane in 3D:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \right) = 0$$

   Which of the following statements are correct?
   A. It is an implicit representation.
   B. It is a slope-intercept representation.
   C. It is a parametric representation.
   D. It is parallel to the vector $(1,1,1)$.
   E. It intersects with the $x-$axis at $(0,0,3)$.
   F. It intersects with the $z-$axis at $(0,0,3)$.
   G. It contains the point $(2,-1,1)$.
   H. It contains the point $(2,-1,2)$.
   I. none of the above

This is a multiple choice question. Answer by marking the correct results.
An explanation is **not** required. In this case, **multiple answers might be correct**.
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■ **Subproblem 2.2 [12 pts]: Planes in 3D**

Assume the vectors $\vec{p}_0 = (1,1,1)$, $\vec{p}_1 = (2,3,3)$, and $\vec{p}_2 = (3,1,1)$ that are representing three points in 3D.

1. Write down the parametric representation of a plane that is defined by these three points. Use $\vec{p}_0$ as support vector in your equation.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

2. Give a normal vector $\vec{n}$ to the plane created above.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

3. Give the implicit representation of the above plane in vector notation.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

4. Give the parametric representation of a plane in 3D that contains the point $(1,2,2)$ and is parallel to the following two lines:

$$l_1(s) = \begin{pmatrix} 1 \\ 4 \\ -4 \end{pmatrix} + s \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix} \text{ and } l_2(t) = \begin{pmatrix} 7 \\ 5 \\ 5 \end{pmatrix} + t \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$$

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

■ **Subproblem 2.3 [8 pts]: Barycentric coordinates**

Assume the following three vectors that represent a triangle in 3D:

$$\vec{a} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix}, \vec{c} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$$

1. Write down how we would represent a point in the barycentric coordinate system defined by this triangle.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

2. What are the conditions that have to be fulfilled if a point that is expressed in barycentric coordinates is within this triangle (including its edges and vertices)?

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

3. Verify if the point $(2,3,3)$ is inside the triangle or not using the conditions from the previous question.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

Problem 3: Matrices and Determinants

■ Subproblem 3.1 [7 pts]: Multiple Choice Questions

1. Let v be a vector and A is a 3×3 matrix, B is a 2×3 matrix, C is a 3×2 matrix, D is a 3×3 matrix. Which of the following calculations are defined?

   A. AB

   B. BA

   C. $ABCv$

   D. $CBAv$

   E. $ABCD$

   F. $DCBA$

   G. none of the above

This is a multiple choice question. Answer by marking the correct results.
An explanation is **not** required. In this case, **multiple answers might be correct**.
Marking wrong answers will slightly reduce your credit but only if your total credit for this question will not be negative.

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2. Which of the following statements are correct, if $A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$.

   A. $\det A = 3$

   B. $\det A = -3$

   C. $\det A^T = 3$

   D. $\det A^T = -3$

   E. none of the above

This is a multiple choice question. Answer by marking the correct results.
An explanation is **not** required. In this case, **multiple answers might be correct**.
Marking wrong answers will slightly reduce your credit but only if your total credit for this question will not be negative.

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3. Which of the following statements are correct, if $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

   A. A is the identity matrix.

   B. A is a diagonal matrix.

   C. A is an orthogonal matrix.

   D. $A = A^{-1}$

   E. $A = B^T$

   F. $A^T + B$ is a diagonal matrix.

   G. none of the above

This is a multiple choice question. Answer by marking the correct results.
An explanation is **not** required. In this case, **multiple answers might be correct**.
Marking wrong answers will slightly reduce your credit but only if your total credit for this question will not be negative.

■ **Subproblem 3.2 [6 pts]: Matrix arithmetics**

- Calculate the result of the following matrix multiplication:

$$\begin{pmatrix} 3 & 1 \\ 4 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ -5 & 2 \end{pmatrix}$$

Answer: If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

- In the tutorials, there was one exercise where we have proven that:

If $AB = AC$, it does ***not*** *necessarily follow that $B = C$.*

However, by using matrix arithmetics, we can get:

$$\begin{array}{rcll} AB & = & AC & \Rightarrow \\ A^{-1}AB & = & A^{-1}AC & \Rightarrow \\ IB & = & IC & \Rightarrow \\ B & = & C & \end{array}$$

Explain what is wrong here. (Hint: what we have proven in the tutorials is correct. One short sentence should be enough to explain the issue.)

Answer: If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

■ **Subproblem 3.3 [7 pts]: Determinants**

1. Compute the determinant of the matrix: $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}$

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

2. Compute the cofactor a^c_{23} of the matrix A above. (Hint: cofactors are mostly used to do Laplace's expansion. You do **not** have to do the full expansion here, but just have to calculate this one single cofactor.)

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

3. Assume two vectors $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$ that are the column vectors of the matrices $M_1 = (\vec{a}\vec{b})$ and $M_2 = (\vec{b}\vec{a})$. Prove that $\det M_1 = -\det M_2$, i.e. that $|\vec{a}\vec{b}| = -|\vec{b}\vec{a}|$.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

■ **Subproblem 3.4 [5 pts]: LESs and Gaussian Elimination**

Assume the following linear equation system (LES):

$$\begin{aligned} x &+ y &+ z &= 4 \\ x &+ 4y &+ 7z &= 16 \\ 2x &+ 2y &+ 4z &= 10 \end{aligned}$$

1. What is the geometric interpretation of this LES?
2. Create the augmented matrix and solve it using Gaussian Elimination.
3. What is the geometric interpretation of your solution?

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

Problem 4: Transformations

■ Subproblem 4.1 [5 pts]: Multiple Choice Questions

1. Assume the following transformation matrix $T = \begin{pmatrix} 1 & 0 & 3 & 3 \\ 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

   The values in the third column represent ...

   A. ...the image of the 2nd base vector under the linear transformation
   B. ...the image of the 2nd base vector under the affine transformation
   C. ...the image of the origin under the linear transformation
   D. ...the image of the origin under the affine transformation
   E. ...none of the above

   The values in the second column represent ...

   A. ...the image of the 2nd base vector under the linear transformation
   B. ...the image of the 2nd base vector under the affine transformation
   C. ...the image of the origin under the linear transformation
   D. ...the image of the origin under the affine transformation
   E. ...none of the above

   The values in the first row represent ...

   A. ...the image of the 2nd base vector under the linear transformation
   B. ...the image of the 2nd base vector under the affine transformation
   C. ...the image of the origin under the linear transformation
   D. ...the image of the origin under the affine transformation
   E. ...none of the above

This is a multiple choice question. Answer by marking the correct result.
An explanation is **not** required. In this case, there is only **one correct answer**.

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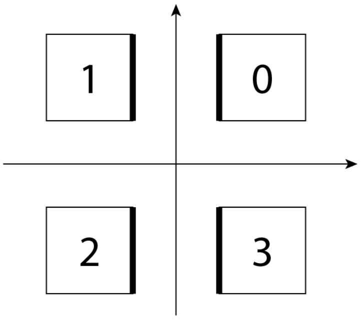
2. The matrix $T = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$ can realize a counterclockwise rotation about the angle ϕ ...

   A. ...around the origin in 2D.
   B. ...around the $x-$axis in 2D.
   C. ...around the $y-$axis in 2D.
   D. ...around the $z-$axis in 2D.
   E. ...around the origin in 3D.
   F. ...around the $x-$axis in 3D.
   G. ...around the $y-$axis in 3D.
   H. ...around the $z-$axis in 3D.
   I. none of the above

This is a multiple choice question. Answer by marking the correct results.
An explanation is **not** required. In this case, **multiple answers might be correct**.
Marking wrong answers will slightly reduce your credit but only if your total credit for this question will not be negative.

■ **Subproblem 4.2 [10 pts]: Finding Transformation Matrices**

1. In the following image, the object on the top right (marked with "0") is transformed by three different transformation matrices T_1, T_2, and T_3. The result of these transformations are illustrated by the three objects marked with "1", "2", and "3", respectively. (Notice that the numbers are just there to mark the object but not part of the actual objects and transformations.)



Write down the three transformation matrices.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

2. Give a matrix for a linear transformation that maps the point $(1,0)$ to $(8,1)$ and the point $(0,1)$ to $(7,3)$.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

■ **Subproblem 4.3 [10 pts]: Linear and Affine Transformations**

1. Give the matrix for uniform scaling with respect to the origin by a factor of 2 in 2D.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

2. Give the matrix for uniform scaling with respect to the point $(1,1)$ by a factor of 2 in 2D.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐

3. Give the matrix for nonuniform scaling with respect to the point $(1,1)$ by a factor of 2 in $x-$direction and a factor of 4 in $y-$direction in 2D.

Answer:

If you need more space to answer this question, make a cross in the box to the right, and continue writing on the separate paper provided by us. ☐