Graphics (INFOGR 2012-2013): Midterm Exam (T1)

Thursday, May 30, 2013, EDUC-GAMMA, 13:30-16:30 (time for the exam: max. 2 hours)

StudentID / studentnummer	Last name / achternaam	First name / voornaam

Do not open the exam until instructed to do so!

Read the instructions on this page carefully!

- You may write your answers in English or Dutch. Use a pen, not a pencil. Do not use red or green.
- Fill in your name and student ID at the top of this page, and on every additional paper you want to turn in.
- Answer the questions in the designated areas on these exam sheets. If you need more space, make a mark at the end of the page and continue writing on the additional paper provided by us. You are not allowed to use your own paper. On the additional paper, make sure to clearly indicate the problem number and don't forget to write your name and student ID on it.
- This is a *closed book exam*. You may **not** use books, notes, or any electronic equipment (including your cellphone, even if you just want to use it as a clock).
- You have max. 2 hours to work on the questions. If you finish early, you may hand in your work and leave, except for the first half hour of the exam. When you hand in your work, have your **student ID** ready for inspection.
- The exam contains 7 problems printed on 13 pages (including this one). It is your responsibility to check if you have a complete printout. If you have the impression that anything is missing, let us know.

Good luck / veel succes!

Please do not write below this line

Problem 1 (max. 20 pts)	
Problem 2 (max. 14 pts)	
Problem 3 (max. 12 pts)	
Problem 4.1 (max. 6 pts)	
Problem 4.2-4.4 (max. 17 pts)	
Problem 5 (max. 8 pts)	
Problem 6 (max. 14 pts)	
Problem 7 (max. 9 pts)	

Points: ______________ Grade: ______________

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

Problem 1: Vectors

■ **Subproblem 1.1 [8 pts]: Multiple choice questions.** Mark the correct answer. No explanation required. There is only one correct answer for each individual question.

1. The two vectors $(3,2,1)$ and $(6,4,2)$ are ...
   - A. parallel
   - B. linearly independent
   - C. unit vectors
   - D. an orthonormal basis
   - E. neither of these
2. The scalar product (aka dot product or inner product) of two perpendicular vectors is ...
   - A. -2π
   - B. -1
   - C. 0
   - D. 1
   - E. 2π
   - F. neither of these
3. The scalar product of two vectors that form an angle of 270 degree with each other is ...
   - A. -2π
   - B. -1
   - C. 0
   - D. 1
   - E. 2π
   - F. neither of these
4. In the following, $\cdot$ denotes scalar multiplication of two vectors and $\times$ denotes their cross product.
   If $\vec{v}, \vec{w}$ are two 3D vectors and s, t are two scalar values, then $(s\vec{v} \times t\vec{w}) + (s\vec{v} \cdot t\vec{w})$ is ...
   - A. undefined
   - B. a scalar value
   - C. a 2D vector
   - D. a 3D vector
   - E. a unit vector
   - F. neither of these

■ **Subproblem 1.2 [7 pts]: Simple vector calculations.**

1. What is the Euclidean length $\|\vec{v}\|$ of the vector $\vec{v} = (0,3,4)$?

   Solution: $\|\vec{v}\| =$

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

2. What is the scalar product $\vec{v} \cdot \vec{w}$ of the two vectors $\vec{v} = (1,2,3)$ and $\vec{w} = (2,2,2)$?

   Solution: $\vec{v} \cdot \vec{w} =$

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3. What is the cross product $\vec{v} \times \vec{w}$ of the two vectors $\vec{v} = (1,2,3)$ and $\vec{w} = (4,1,1)$?

   Solution: $\vec{v} \times \vec{w} =$

■ **Subproblem 1.3 [5 pts]: Vector characteristics.**

Prove that $\lambda(-y,x)$ is a normal vector to (x,y) for all $\lambda \neq 0$.

Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

Problem 2: Basic geometric entities

■ **Subproblem 2.1 [6 pts]: Multiple choice questions.** Mark the correct answer. No explanation required. There is only one correct answer for each individual question.

1. If $y = ax + c$ denotes the *slope-intercept form* of a line in 2D, then c gives us ...
   A. the slope of the line
   B. the fraction of the slope in $X-$direction
   C. the fraction of the slope in $Y-$direction
   D. the intersection of the line with the $X-$axis
   E. the intersection of the line with the $Y-$axis
   F. neither of these
2. If $2x - 3y + 5 = 0$ denotes the *implicit representation* of a line in 2D, then the vector $(2, -3)$ is ...
   A. a vector on the line
   B. a vector pointing to the line
   C. a vector perpendicular to the line
   D. a vector parallel to the line
   E. neither of these
3. If $\vec{p}(t) = (2,3) + t(4,1)$ denotes the *parametric equation* of a line in 2D, then the vector $(2,3)$ is ...
   A. the line's support vector
   B. the line's direction vector
   C. a normal to the line
   D. a vector parallel to the line
   E. neither of these

■ **Subproblem 2.2 [8 pts]: Basic geometric entities in 3D.**

1. Write down the *parametric equation* of a plane in 3D that goes through the three points represented by the vectors $\vec{p}_0 = (1,2,3)$, $\vec{p}_1 = (3,3,3)$, and $\vec{p}_2 = (3,2,3)$. Use $\vec{p}_0$ as support vector of the plane.

   Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

2. Write down the *implicit equation* of a plane in 3D in vector notation, i.e. $\vec{n} \cdot (\vec{p} - \vec{p}_0) = 0$ that contains the point $(1,1,1)$ and is parallel to the two vectors $\vec{v} = (1,2,3)$ and $\vec{w} = (4,1,1)$. (Hint: it is allowed to reuse results from previous problems.)

   Answer:

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3. The following represents the *parametric equation* of a sphere around the origin in 3D:

$$\vec{p}(\Phi, \Theta) = \begin{pmatrix} 5\cos\Phi\sin\Theta \\ 5\sin\Phi\sin\Theta \\ 5\cos\Theta \end{pmatrix}$$

   Write down the parametric equation of a sphere around the point $\vec{c} = (2,3,1)$ with the same radius.

   Answer:

---

4. The following represents the *implicit equation* of a sphere around the origin in 3D:

$$x^2 + y^2 + z^2 - 9 = 0$$

   Write down the implicit equation of a sphere around the point $\vec{c} = (2,3,1)$ with the same radius.

   Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

Problem 3: Intersections

■ **Subproblem 3.1 [6 pts]: Multiple choice questions.** No explanation required.

1. When calculating the intersection of two lines in 3D, which of the following can be a possible outcome? Multiple answers might be correct. You only get credit for this subquestion if you mark *all* correct ones and none that is incorrect.

   A. no solution B. a point C. a line D. a plane E. a circle F. a triangle

2. When calculating the intersection of two spheres in 3D, which of the following can be a possible outcome? Multiple answers might be correct. You only get credit for this subquestion if you mark *all* correct ones and none that is incorrect.

   A. no solution B. a point C. a line D. a plane E. a circle F. a triangle

3. Assume a point in 3D in *barycentric coordinates*, i.e. $\vec{p}(\beta,\gamma) = \vec{a} + \beta(\vec{b} - \vec{a}) + \gamma(\vec{c} - \vec{a})$ with $\beta \geq 0$ and $\gamma \geq 0$. What other condition has to be fulfilled if the point is within the triangle defined by the three points $\vec{a}$, $\vec{b}$, $\vec{c}$? There is only one correct answer to this question.

   A. $\beta \leq 1, \gamma \leq 1$
   B. $\beta + \gamma \leq 1$
   C. $\beta + \gamma = 1$
   D. $\frac{1}{2}(\beta + \gamma) \leq 1$
   E. $\frac{1}{2}(\beta + \gamma) = 1$
   F. neither of these

■ **Subproblem 3.2 [6 pts]: Intersection of lines.**

Assume two lines in 2D: line l_1 is defined by the two points $(1,2)$ and $(3,3)$ and line l_2 is defined by the two points $(1,1)$ and $(3,2)$. Do these two lines intersect? If yes, calculate the intersection point(s). If no, explain why.

Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

Problem 4: Matrices and determinants

■ **Subproblem 4.1 [6 pts]: Multiple choice questions.** No explanation required.
There is only one correct answer for each individual question.

1. Which of the following statements is correct? If A is a diagonal matrix, then ...
   - A. $A^T = I$
   - B. $A^T = A$
   - C. $A^T = A^{-1}$
   - D. $A^{-1} = I$
   - E. A^{-1} is always undefined
   - F. neither of these
2. Which of the following statements is **not** true for any three random matrices A, B, and C with the same dimensions?
   - A. $(AB)C = A(BC)$
   - B. $AB = BA$
   - C. $A(B+C) = AB + AC$
   - D. $(A+B)C = AC + BC$
3. Assume we have a 2×3 matrix A, a 2D vector $\vec{v}$, a 3D vector $\vec{w}$, and $\cdot$ denotes matrix multiplication.
   What is the result of $(\vec{v} \cdot A \cdot \vec{w}^T) + (\vec{w} \cdot A^T \cdot \vec{v}^T)$?
   - A. a 3×1 matrix
   - B. a 2×1 matrix
   - C. a 1×1 matrix
   - D. a 1×2 matrix
   - E. a 1×3 matrix
   - F. neither of those

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

■ **Subproblem 4.2 [4 pts]: Simple matrix calculations.**

Assume the following two matrices: $A = \begin{pmatrix} 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ 1 & 2 \end{pmatrix}$.

1. Calculate the product AB using matrix multiplication.

   Answer:

---

2. Calculate the sum $A + B^T$.

   Answer:

■ **Subproblem 4.3 [5 pts]: Matrix characteristics.**

Assume a 2×2 matrix A where the two column vectors are parallel to each other.
Prove that the determinant $\det A$ of this matrix is zero.

Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

■ **Subproblem 4.4 [8 pts]: Determinants.**

1. Complete the following sentence in a way that it becomes a correct statement.

   The determinant of a 2×2 *matrix is the* ______________________________ *area*

   of the ______________________________ *defined by its two column vectors.*

2. Calculate the determinant $\det A$ of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

   Answer:

3. Calculate the cofactor a^c_{12} of the matrix A from the previous question.
   (Note: We start counting rows and columns with 1, so a_{12} is the coefficient in the 1st row and 2nd column.)

   Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

Problem 5: Linear equation systems

■ **Subproblem 5.1 [2 pts]: Geometric interpretation.** Assume you are using Gaussian elimination to solve a linear equation system with three variables and three equations. You end up getting exactly one line with $0x+0y+0z=0$. What is the geometric interpretation of this outcome?

Answer:

■ **Subproblem 5.2 [6 pts]: Gaussian elimination.** Assume the following linear equation system:

$$\begin{array}{rrrl} x & +2y & +2z & =5 \\ x & +4y & +2z & =9 \\ -x & -2y & & =1 \end{array}$$

1. Write down its augmented matrix and solve it using Gaussian elimination.

   Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

2. What is the geometric interpretation of your result?

   Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

Problem 6: Transformations

■ **Subproblem 6.1 [4 pts]: Linear transformations.**

1. Assume the following matrix for linear transformations in 3D:

$$A = \begin{pmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   What kind of transformation do we get if we apply it to a vector in 3D?

   Answer:

2. Write down the transformation matrix for the inverse transformation of A (i.e. a matrix that transforms a vector $\vec{v}_t = A\vec{v}$ back to vector $\vec{v}$).

   Answer:

■ **Subproblem 6.2 [10 pts]: Affine transformations.**

1. Assume the following matrix for affine transformations in 3D:

$$\begin{pmatrix} 3 & 0 & 0 & 2 \\ 0 & 3 & 0 & 2 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   What kind of transformation do we get if we apply it to a vector in 3D?

   Answer:

2. Complete the following sentences in a way that creates a correct statement.
   In a transformation matrix for affine transformations in 3D . . .

   - . . . the last column
     represents the image of ____________________
   - . . . the first 3 columns
     represent the image of ____________________
   - . . . the coordinates
     in the last row are called ____________________

Make a mark here if you answered parts of these questions on an extra sheet: [ ]

Problem 7: Texturing

■ **Subproblem 7.1 [4 pts]: Multiple choice questions.** No explanation required.
There is only one correct answer for each individual question.

1. Which of the following statements is correct?

   The Hermite interpolation used to create Perlin noise uses ...

   A. no weights B. linear weights C. quadratic weights D. cubic weights E. sine wave weights

2. Which of the following statements is **not** correct?

   Bump mapping ...

   A. is used for shading calculations.
   B. uses an array of vectors.
   C. causes an actual change of the geometry.
   D. usually requires less storage than displacement mapping.
   E. usually produces worse results than displacement mapping.

■ **Subproblem 7.2 [2 pts]: Procedural texturing.**

Fill in the blank space in the following piece of pseudo code so that we can use it to create a stripe pattern with stripes of width π and alternating colors `color0` and `color1` along the $Z-$axis. The resulting strip pattern should have a color change at the origin.

```
stripe(x_p, y_p, z_p){

  if (____________________)
    return color0;
  else
    return color1;
}
```

■ **Subproblem 7.3 [3 pts]: Perlin noise.**

In the standard approach to calculate Perlin noise, we create random vectors (v_1, v_2, v_3) with $-1 \leq v_i \leq 1$ for $i = 1, 2, 3$. Yet, we only use the ones for which $(v_1^2 + v_2^2 + v_3^2) < 1$. Shortly explain why.
(Note: a short explanation is sufficient. One sentence can be enough to get full credit for this subproblem.)

Answer:

Make a mark here if you answered parts of these questions on an extra sheet: [ ]