Graphics (INFOGR 2017-2018) – Midterm Exam

Tuesday May 22nd, 08.30 – 10.30 – EDUC-BETA

- Write your answers, along with solution steps, on the supplied answer sheets.
- State your name and student ID at the top of every answer sheet you want to turn in.
- **Write clearly:** we cannot allocate points for answers that we cannot read.
- No documents allowed. Use of all electronic devices is forbidden.
- If a question is unclear to you, write down how you interpret the question, then answer it.
- The font used for this exam is OpenDyslexic, for your comfort.

PART 1 – MATH - max 36 points

1. [2+5=7 points] Given are two points: $P = (1,2,3)$ and $Q = (5,10,11)$ in $\mathbb{R}^3$, which lie on line L.

    a. Write down the general implicit equation of a plane perpendicular to line L.
    $$x + 2y + 2z + D = 0$$

    b. We draw a line from point $R = (3,8,5)$ that is perpendicular to line L, intersecting it at point S. Calculate the length of line segment RS. $\quad \sqrt{8} = 2\sqrt{2}$

2. [3+3=6 points] Consider three points in $\mathbb{R}^2$: $A = (1,1)$, $B = (-3,4)$ and $C = (1,7)$.

    a. We place a light at point C. What is the length of the shadow of the line segment AB on the x-axis? $\quad \frac{28}{3} = 9\frac{1}{3}$

    b. We place a camera at point B, viewing line segment AC, rendering it on the y-axis as the one-dimensional 'screen' as $A'C'$. What is the length of the line segment $A'C'$? $\quad 9/2 = 4.5$

3. [1+5+3=9 points] Given: a sphere in $\mathbb{R}^3$, with centre $C = (3,3,3)$ and a point on the surface of the sphere: $P = (2,5,1)$.

    a. Write down the implicit eq. for the sphere. $\quad (x-3)^2 + (y-3)^2 + (z-3)^2 = 9$

   b. Calculate the point on the surface of the sphere closest to $(6,9,1)$.
    $$(30/7, 39/7, 15/7)$$

    c. Unit vector $\hat{u} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ is a tangent vector of the sphere at point P. Calculate the bitangent vector of the sphere at point P. $\quad \left[\frac{2\sqrt{2}}{3}, \frac{1}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}}\right]$

4. [4 points] We define a coordinate system in $\mathbb{R}^2$ (i.e., x- and y-axes and the origin). Draw this coordinate system and shade the region for which two conditions hold: $x + y > 1$ and $x + 1 < y$.
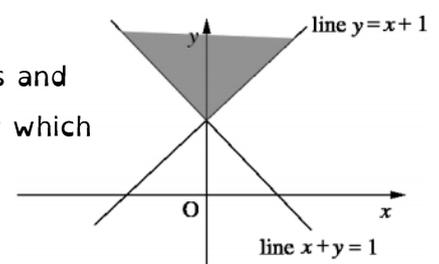
5. [3 points] Write down the implicit equation of the tangent plane
to the sphere $(x-3)^2 + (y-4)^2 + z^2 = 9$ at point $P = (5,5,2)$. **$2x + y + 2z - 19 = 0$**

6. [2+1+4=7 points] Consider *Figure 1* below, which depicts a situation in $\mathbb{R}^2$. Given:

   ▪ Line P, defined as $x - 2y + 1 = 0$ and line Q, defined as $y - 2x - 3 = 0$
   ▪ Points A and B on line Q. The location of A is (0,3). The length of line AB is w.
   ▪ The points A and B are projected onto line P at A' and B' respectively, i.e. AA' and BB' are both perpendicular to line P.
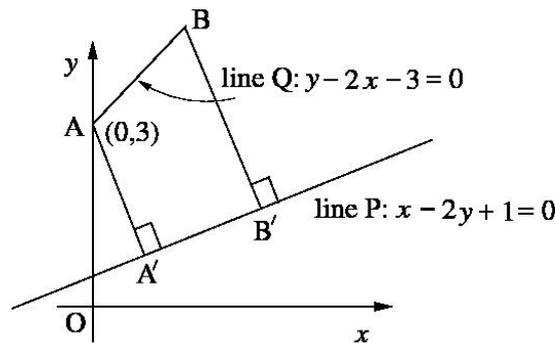
   a. Calculate the length of line segment AA'. **$\sqrt{5}$**

   b. Determine the location of point A'. **$(1,1)$**

   c. Express the length of $A'B'$ as a function of w. **$4w/5$**

PART 2 – THEORY - max **10** points

7. [6 points] A texture is stored as a palettized image. The dimensions of the texture are 512 x 512 pixels, and it uses exactly 256 unique colors. How much memory (in bytes) is needed to store this texture? **$512^2 + 1024$ or $512^2 + 768$**

8. [4 points] Complete the following sentence. Write down the four terms that complete the sentence on your answer sheet.

   "The flickering and Moiré-patterns we see on distant textured objects are symptoms of <u>UNDERSAMPLING</u>. This problem can be reduced by using <u>MIPMAPPING</u> . When a textured object is close to the camera, the texture may appear blocky. This is caused by <u>OVERSAMPLING</u>. We can smooth out the blocky texture using <u>BILINEAR INTERPOLATION</u>.

   Note: only the actual terms are allowed; descriptions score no points.