INFOGR 2020 - Midterm
Duration: 120 minutes + 10 minutes submission period
Total points: 61

Please read the following remarks carefully: *This is an **open book exam**. The use of electronic devices, such as calculators, smartphones, or laptops is allowed but will not be necessary for a passing grade.*

*You will work on this midterm and **submit on an individual basis**. We will follow the standard Utrecht University exam regulations and check the submissions for plagiarism.*

During the midterm several supervisors will available for understanding questions.

*The midterm contains **seven tasks**. Do consider the point distribution for an efficient/effective work.*

*In your answers, please try to be brief and **focus on the key aspects**. Where useful, please use bullet lists and annotated sketches to provide for compact answers.*

Duration: *120 minutes + 10 min submission period. Bear in mind that the submission process takes some time. However, late submissions will receive a point deduction (-1P each 2 minutes).*

Extension Time: *If you are eligible for an extension time (according to the UU regulations), make a note of this on your answer sheet. You can hand in 20 minutes late (plus the regular 10 minutes for the submission process). All claims will be checked.*

Submission Agreement: *With your submission (sending an email to infogrUU@gmail.com) you declare that you produced the results of this exam yourself, without the direct help of others persons. If the examiner is unsure whether an answer has been reached without assistance, he can invite the student to elaborate the results per MS Teams, Skype or another video-capable conferencing tool.*

Submission Process: *(0) It is your responsibility to submit a gradable and readable submission. (1) Put page numbers on each of your sheets. (2) Take photos of the final submission sheets (You may use tools like CamScanner (or similar) to hand in your work, but a photo upload is generally enough). (3) Use your favorite email client to send an email submission to infogrUU@gmail.com. (4) Use the **attachment functionality for all photos**. (5) Before sending, verify the readability and completeness of your submission. (6) click on 'SEND' after you are done (We will grade your first submission only).*

Question 1. [**1 point each = 9 points**] Answer the following understanding questions with a precise and concise explanation. You can use *annotated* sketches if necessary.

(a) Explain in your own words the difference between a scalar, a point and a vector, using an example in daily life.
(b) Describe the term unit vector.
(c) Write down and explain with an annotated sketch the Pythagoras' theorem.
(d) What is a 'orthogonal basis' for a coordinate system?
(e) For what angle between two vectors is the dot product of those vectors at its largest?
(f) What is the relation between the magnitude of a vector and the dot product of the vector with itself?
(g) Write down a general line in the slope-intercept form. What is the meaning of every term?
(h) Explain in your own words the geometric interpretation of a cross product of two vectors (in 3D).
(i) Is the dot product a vector or a scalar? And what about the cross product?

Question 2. [**2+5=7 points**] Consider two points $P = (3, 2)$ and $Q = (1, 4)$ in $\mathbb{R}^2$. Please answer (and *outline the intermediate steps*) for the questions below.

(a) Write down the equation of the line passing through them in implicit form.
(b) The line segment PQ is one arm of a full square $PQRS$; the vertices are labelled in the clockwise direction. Find the coordinates of R and S.

Question 3. [**4+2+3=9 points**] Given are the points in 3D: $A = (2, -1, -2)$, $B = (3, 1, -1)$ and $C = (1, -1, -1)$. Please answer (and *outline the intermediate steps*) for the questions below.

(a) Write down the general form of the implicit equation of the plane P through A, B and C.
(b) Determine the unit vectors perpendicular to P.
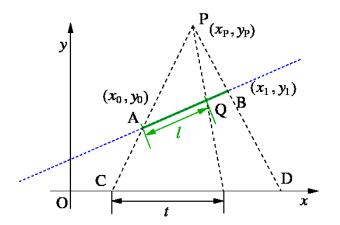(c) What is the minimal distance of point $M = (5, 5, 5)$ to the plane P?

Question 4. [**2+7=9 points**] Given a sphere in $\mathbb{R}^3$ with centre C $= (3, 3, 3)$ and a point on the surface of the sphere P $= (2, 5, 1)$. Please answer (and *outline the intermediate steps*) for the questions below.

(a) Determine the equation for the sphere in implicit and parametric form.
(b) Determine the location of the point on the surface of the sphere closest to Q $= (6, 9, 1)$.

Question 5. [**2+2+2=6 points**] Given a point $P = (3, 4)$ and a circle centered around P with radius 2. Also consider two points $A = (-2, 1)$ and $B = (5, 6)$. Please answer (and *outline the intermediate steps*) for the questions below.
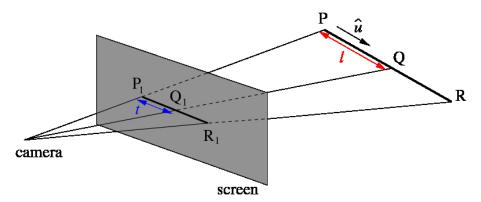
(a) Give the equation of the circle in implicit and parametric form.
(b) Determine the equation for line l through A and B in slope-intersect form.
(c) Write down the coordinates of one point of the intersection of l with the circle in question (a).

Question 6. [**2+1+4+3=10 points**] Given a set of points A, B, C, D, Q and P as shown in the figure below (at $P = (4, 5)$ there is a light source, and the shadows of A and B on the x-axis are C and D respectively). A line k passes through A, Q and B. Please answer (and *outline the intermediate steps*) for the questions below.

(a) Given that $A = (3, 2)$ and $B = (5, 3)$, give the equation for line k through A and B in the implicit and parametric form.

(b) If Q has x-coordinate 4. Determine its y-coordinate.

(c) Determine the coordinates of C and D.

(d) Determine t as a function of l (Note that Q is not fixed anymore, as in question [b]).

Question 7. [**6+5=11 points**] Given two points $P = (2, 3, 4)$ and $R = (5, 6, 4)$, and camera at point $E = (3, 2, -6)$. The xy-plane is the screen. Please answer (and *outline the intermediate steps*) for the questions below.

(a) Project PR to $P_1 R_1$ on the screen as seen by the camera (see figure). Obtain the coordinates of P_1 and R_1.

(b) Given $P_1 Q_1 = t$, calculate the coordinates of point Q.

3