Graphics (INFOGR 2017/2018) – Final Exam

Thursday June 28th, 13.30 – 15.30 – EDUC-BETA / BBG 0.23 (ET)

- Write your answers in the designated areas on the question sheet. There's also some room to write down the steps involved. If you need more space, please use the blank answer sheets supplied to you.
- If a question is unclear to you, write down how you interpret the question, then answer it.
- **Write clearly:** we cannot grade answers that we cannot read.
- No documents allowed. Use of electronic devices is forbidden.
- You may use a dictionary, and answer in English or Dutch.

	1	2	3	4	5	6	7
a							
b							
c	for grading, leave blank						
d							
e							

StudentID / studentnummer	Last name / achternaam	First name / voornaam

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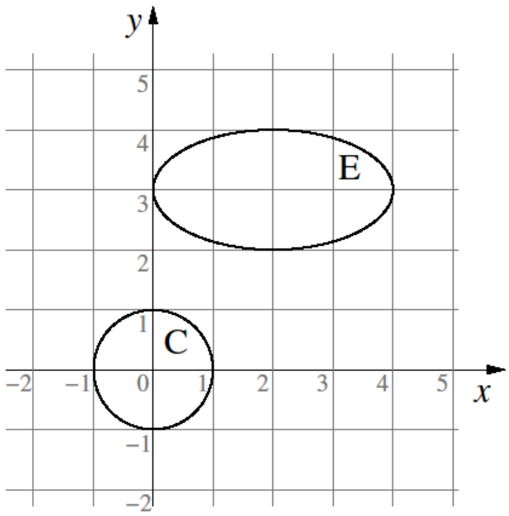
PART 1 – MATH - max 43 points

1. [2+3=5 points] We want to transform the ellipse E into circle C using a transformation matrix; see figure.



a. Write down the elementary active transformations needed for this transformation.

Transformations:

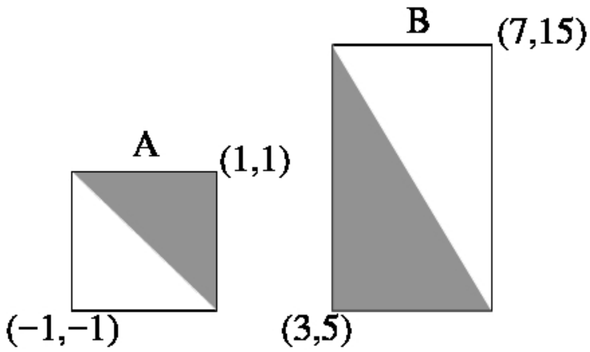
- Scaling
- Translation

Note: Primary solution for C→E, secondary for E→C.

b. Write down the final transformation matrix M.

$$M = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1/2 & 0 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

2. [3+5=8 points] We want to transform square A to rectangle B, as shown in the figure (figure not to scale).



a. Write down the elementary active transformations that you need to achieve this.

Transformations:

- 180 degree rotation
- Scaling
- Translation *(order is relevant, but not for your score – covered in 2b)*

b. Write down the final transformation matrix M.

$$M = \begin{bmatrix} -2 & 0 & 5 \\ 0 & -5 & 10 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. [1+3=4 points] Consider the two points $P = (1,1)$ and $Q = (4,5)$ in $\mathbb{R}^2$. Consider also the point $X = (x, y)$ such that the lines XP and XQ intersect each other at right angles (i.e., the lines are perpendicular).
The collection of all such points X that satisfy the above condition forms a primitive. Name the primitive and write down the equation for this primitive.

The primitive is a (or an): CIRCLE (technically ellipse is also correct)

Equation: $\left(x - \frac{5}{2}\right)^2 + (y-3)^2 = \frac{25}{6} = 6\frac{1}{4}$.

The dot product of the vectors PX and QX is zero, which yields

$$(x-1)(x-4) + (y-1)(y-5) = 0.$$

4. [3+3+5+3+2=16 points] Consider the plane $x + 2y + 2z = 9$ and point $O = (0,0,0)$ in $\mathbb{R}^3$.

a. Project O on the plane. Write down the coordinates of the projected point O'.

$O' = (\ 1 \ , \ 2 \ , \ 2 \ )$

b. Reflect O in the plane. Write down the coordinates of the reflected point R.

$R = (\ 2 \ , \ 4 \ , \ 4 \ )$

Solution: Shoot a ray from the origin along $\hat{n} = \frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}$. The equation of this ray is $\begin{bmatrix}x\\y\\z\end{bmatrix} + t\hat{n}$. For the projected point, which lies on this line, we get $t = 3$, leading to the answer above. To obtain the reflected point we merely double that value, i.e., use $t = 6$.

c. Reflect the vector $\vec{v} = \begin{bmatrix}v_x\\v_y\\v_z\end{bmatrix}$ in this plane using a transformation matrix. Write down the transformation matrix M_r and the reflected vector $\vec{v}_r$.

$$M_r = \frac{1}{9}\begin{bmatrix}7 & -4 & -4\\-4 & 1 & -8\\-4 & -8 & 1\end{bmatrix} \qquad \vec{v}_r = M_r\vec{v} = \frac{1}{9}\begin{bmatrix}7v_x - 4v_y - 4v_z\\-4v_x + v_y - 8v_z\\-4v_x - 8v_y + v_z\end{bmatrix}$$

d. Reflect the point $P = (x, y, z)$ in this plane to obtain the coordinates of the reflected point $P' = (x', y', z')$.

$x' =$ 7x/9 – 4y/9 – 4z/9 + 2

$y' =$ -4x/9 + y/9 – 8z/9 + 4

$z' =$ -4x/9 – 8y/9 + z/9 + 4

Solution: The vector from the from origin to the reflected origin is given by $\begin{bmatrix}2\\4\\4\end{bmatrix}$. Draw the vector $\begin{bmatrix}x\\y\\z\end{bmatrix}$ from the origin to the point P. Following (b), the reflected vector is given by $\begin{bmatrix}7x/9 - 4y/9 - 4z/9\\-4x/9 + y/9 - 8z/9\\-4x/9 - 8y/9 + z/9\end{bmatrix}$. Adding the two we obtain the co-ordinates of the reflected point P′.

e. Write down transformation matrix M that directly transforms P to P'.

$$M = \begin{bmatrix}7/9 & -4/9 & -4/9 & 2\\-4/9 & 1/9 & -8/9 & 4\\-4/9 & -8/9 & 1/9 & 4\\0 & 0 & 0 & 1\end{bmatrix}$$

5. [4+(3+3)=10 points] A camera is located at point $P = (3,4,5)$ in world space, gazing in direction $\hat{g} = -\frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}$. The "view up vector" for the camera is $\hat{y}_c = \frac{1}{3}\begin{bmatrix}-2\\2\\-1\end{bmatrix}$.

   a. Determine the unit vectors $\hat{x}_c$ and $\hat{z}_c$ that together with $\hat{y}_c$ define the camera coordinate system.

$$\hat{x}_c = \frac{\hat{t}\times\hat{w}}{||\hat{t}\times\hat{w}||} = \frac{1}{3}\begin{bmatrix}2\\1\\-2\end{bmatrix}, \hat{z}_c = -\hat{g} = \frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}$$

   b. Write down the two transformation matrices that are required to transform from world space to camera space, in the correct order.

$$M = \begin{bmatrix}2/3 & 1/3 & -2/3 & 0\\-2/3 & 2/3 & -1/3 & 0\\1/3 & 2/3 & 2/3 & 0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & -3\\0 & 1 & 0 & -4\\0 & 0 & 1 & -5\\0 & 0 & 0 & 1\end{bmatrix}$$

PART 2 – THEORY - max 12 points

6. [3+3=6 points] In a rasterizer, we can use environment mapping to make objects appear reflective. Write down two limitations of this technique.

1: An envmap reflects static surroundings (no animation)

2: The envmap is a bitmap (lack of detail / pixelated)

3: The envmapped object cannot reflect itself

Note: true but irrelevant statements score no points. Repeating one statement in different words scores only 3 points (if correct).

7. [max 6 points] A triangle has three vertices, located in screen space at $(-1,5)$, $(2,2)$ and $(3,9)$. Clip this triangle against the line $x = 0$ using the Sutherland-Hodgeman algorithm. Write down the emitted coordinates for each edge of the input triangle.

Edge 1: (going in) emit (0,4) and (2,2)

Edge 2: (staying in) emit (3,9)

Edge 3: (going out) emit (0,6)

Grading: 1 point per correct edge, 3 extra for a complete solution. Clipping as if x<0 is 'in' is also fine, if done correctly. Reversing vertex order or a different vertex as a start point is also fine.

That's all, good luck!