Final Test
Motion and Manipulation

November 11, 2011
13:30-15:30

Note: It is not allowed to use pocket calculators or consult books, notes, slides, etc. Fill out your name and student number on each page you hand in. The test consists of **nine** exercises. **Motivate all your answers!**

1: Geometric Modeling (1.0)

Let $p_1 = (3,0)$, $p_2 = (3,5)$, $p_3 = (0,2)$, $p_4 = (-3,5)$, and $p_5 = (-3,0)$. Consider the non-convex object M bounded by the edges p_1p_2, p_2p_3, p_3p_4, p_4p_5, and p_5p_1. Define the object M as a union of intersections of half-planes $H_i = \{ (x,y) \in \mathbb{R}^2 \mid f_i(x,y) \leqslant 0 \}$.

2: Configuration Spaces (1.0)

Determine the configuration space for a system of two ball-shaped robots A_1 and A_2 moving in contact and a cube-shaped robot A_3 moving independently (from A_1 and A_2) in a three-dimensional Euclidean workspace.

3: Minkowski Sums (1.0)

Let $p_1 = (1,1)$, $p_2 = (5,3)$, $p_3 = (1,5)$, and $p_4 = (3,3)$. Let V be the non-convex object bounded by the edges p_1p_2, p_2p_3, p_3p_4, and p_4p_1. Let S be the square with corners $(0,0)$, $(2,0)$, $(2,2)$, and $(0,2)$. Construct the Minkowski sum $V \oplus S$ and report its vertices.

4: Kinematics (1.0)

We are given a fixed orthonormal frame $F = \{f^1, f^2, f^3\}$ and a mobile orthornormal frame $M = \{m^1, m^2, m^3\}$. Initially the frames M and F coincide. We translate M along m^2 by 3 units, and then rotate M about m^1 by $\pi/6$ radians. Determine the homogeneous transformation matrix that maps mobile M coordinates into fixed F coordinates. Transform the M coordinates $(0,0,2)$ into F coordinates.

5: Kinematics for Linkages (1.0)

Consider the four-axis robot on the separate sheet and the frames assigned to its axes of motion and its hand. Use the given frames to determine the joint angle θ_i, the joint distance d_i, the link length a_i, and the link twist angle α_i for each of the axes $i = 1,2,3,4$. Clearly indicate which parameters are variable.

6: Path Planning (2.0)

Consider a point robot moving in a two-dimensional Euclidean workspace amidst a collection of non-intersecting polygonal obstacles with a total of n vertices. Describe how to obtain the trapezoidal decomposition of this workspace with obstacles. What is the complexity of the resulting decomposition? Describe how the decomposition is subsequently processed into a graph that facilitates path planning queries. Finally describe how the decomposition and graph can be used to solve a path planning query between an initial configuration q and a goal configuration q'.

7: Plücker Coordinates (1.0)

Determine the Plücker coordinates of the line ℓ through the points $(2,3,4)$ and $(5,6,0)$. What is the distance from ℓ to $(0,0,0)$?

8: Grasping and Form Closure (1.0)

Let $p_1 = (0,0)$, $p_2 = (4,-2)$, $p_3 = (0,6)$, and $p_4 = (-4,-2)$. Let N be the non-convex object bounded by the edges p_1p_2, p_2p_3, p_3p_4, and p_4p_1. Assume that a frictionless point finger is placed at p_1. Place the smallest number of additional frictionless point fingers to put N in form closure. Use half-plane analysis of instantaneous velocity centers to verify your answer.

9: Grasping and Force Closure (1.0)

Consider the convex object C defined by the vertices $p_1 = (2,-3)$, $p_2 = (2,2)$, $p_3 = (-2,2)$, $p_4 = (-2,-3)$, and $p_5 = (0,-5)$. Consider three frictionless point fingers placed at $a_1 = (2,1)$, $a_2 = (-1,2)$, and $a_3 = (-2,-1)$. Prove that no placement of a fourth finger at a point a_4 on the edge p_4p_5 will yield force closure for C by the fingers at a_1, a_2, a_3, and a_4.