Final Test
Motion and Manipulation

November 6, 2013
13:30-16:00

Note: It is not allowed to use pocket calculators or consult books, notes, slides, etc. Fill out your name and student number on each page you hand in. The test consists of **ten** exercises. **Motivate all your answers!**

1: Kinematics I (1.0)

We are given a fixed orthonormal frame $F = \{f^1, f^2, f^3\}$ and a mobile orthornormal frame $M = \{m^1, m^2, m^3\}$. Initially the frames M and F coincide. We rotate M by an angle $\pi/6$ about a line through the origin with global direction vector $(\frac{1}{2}\sqrt{2}, \frac{1}{3}\sqrt{3}, \frac{1}{6}\sqrt{6})$. Determine the rotation matrix R that maps coordinates with respect to M to coordinates with respect to F.

2: Kinematics II (1.0)

We are given a fixed orthonormal frame $F = \{f^1, f^2, f^3\}$ and a mobile orthornormal frame $M = \{m^1, m^2, m^3\}$. Initially the frames M and F coincide. We translate M along m^2 by 3 units, and then rotate M about m^1 by $\pi/6$ radians. Determine the homogeneous transformation matrix that maps mobile M coordinates into fixed F coordinates. Transform the M coordinates $(0, 0, 2)$ into F coordinates.

3: Collision Detection (1.0)

Let S be the square with corners $p_1 = (4, 2)$, $p_2 = (4, 10)$, $p_3 = (-4, 10)$, and $p_4 = (-4, 2)$. Demonstrate how the GJK algorithm proceeds to find the distance from the origin O to S, starting from the initial inscribed simplex with corners p_1, p_2, and p_3.

4: Configuration Spaces (0.5 + 0.5)

(a.) Determine the configuration space for a system of two ball-shaped robots A_1 and A_2 moving in contact and a cube-shaped robot A_3 moving independently (from A_1 and A_2) in a three-dimensional Euclidean workspace.

(b.) Consider a two-dimensional Euclidean workspace with a line-segment robot A with endpoints $(0, 0)$ (its reference point) and $(1, 1)$ and a disk-shaped obstacle $D = \{(x, y) \mid x^2 + y^2 - 1 \leqslant 0\}$. The robot A is only allowed to translate. Construct the configuration-space obstacle C_{obs} corresponding to all placements in which A intersects O.

5: Kinematics for Linkages (1.0)

Consider the four-axis robot on the separate sheet and the frames assigned to its axes of motion and its hand. Use the given frames to determine the joint angle θ_i, the joint distance d_i, the link length a_i, and the link twist angle α_i for each of the axes $i = 1, 2, 3, 4$. Clearly indicate which parameters are variable.

6: Short Questions (0.5 + 0.5)

Give short answers to each of the following questions.

(a.) What is *open-loop control*?

(b.) What is an *actuator*?

7: Manipulation (1.0)

Construct a convex polygonal object O with four vertices such that the number of stable orientations when O is squeezed by two parallel jaws is as small as possible.

8: Representation of Lines (1.0)

What is the distance from $(0, 0, 0)$ to the line ℓ through the points $(0, 1, 3)$ and $(1, 2, 1)$?

9: Form Closure Grasps (1.0)

Use arguments based on half-plane analysis of velocity centers to show that a disk $D = \{(x, y) \mid x^2 + y^2 - 1 \leqslant 0\}$ cannot be put in form closure with any number of frictionless point fingers.

10: Force Closure Grasps (1.0)

Let $p_1 = (0, 0)$, $p_2 = (3, -3)$, $p_3 = (3, 1)$, $p_4 = (-3, 1)$, and $p_5 = (-3, -3)$. Let N be the non-convex object bounded by the edges p_1p_2, p_2p_3, p_3p_4, p_4p_5, and p_5p_1. Assume that a frictionless point finger is placed at p_1. Place the smallest number of additional frictionless point fingers to put N in force closure. Prove that the resulting grasp puts N in force closure.