Final Test
Motion and Manipulation

November 4, 2014
17:00-19:30

Note: It is not allowed to use pocket calculators or consult books, notes, slides, etc. Fill out your name and student number on each page you hand in. The test consists of **eight** exercises. **Motivate all your answers!**

1: Kinematics (1.0)

Determine the angle and axis of rotation corresponding to the rotation matrix

$$R = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{2} & -\frac{1}{2} \\ -\frac{1}{2}\sqrt{2} & 0 & -\frac{1}{2}\sqrt{2} \\ -\frac{1}{2} & \frac{1}{2}\sqrt{2} & \frac{1}{2} \end{pmatrix}.$$

2: Inverse Kinematics (2.0)

We consider a two-link robot in two-dimensional Euclidean space with two rotational degrees of freedom. It consists of a link of length 4 and a link of length 3 that share a common endpoint that acts as a hinge. The other endpoint of the long link is anchored at the origin about which the link can rotate. The free endpoint of the short link is the *tip* of our robot. The relation between the position (x, y) of the tip and the joint angles θ_1 and θ_2 is now given by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4\cos\theta_1 + 3\cos(\theta_1 + \theta_2) \\ 4\sin\theta_1 + 3\sin(\theta_1 + \theta_2) \end{pmatrix}.$$

The goal is to determine values for θ_1 and θ_2 that place the tip at $(5, 4)$. We consider the use of the iterative solution method to this inverse kinematics problem. Perform one iteration of the iterative solution method to find improved values $\theta_1^{(1)}$ and $\theta_2^{(1)}$ for θ_1 and θ_2 respectively, using initial guesses $\theta_1^{(0)} = 0$ and $\theta_2^{(0)} = \pi/2$.

3: Line Representations (1.0)

Consider the lines

$$l_c : \overline{x} = \begin{pmatrix} 2 \\ c \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

for $c \in \mathbb{R}$. Is there a line l_c with a distance smaller than 1 to $(0, 0, 0)$?

4: Minkowski Sums (1.0)

Let s_0 be the line segment with endpoints $(1,0)$ and $(4,0)$ and s_1 be the line segment with endpoints $(1,0)$ and $(1,1)$ and define $L = s_0 \cup s_1$. Let T be the triangle with corners $(1,1)$, $(4,1)$, and $(1,3)$. Construct the Minkowski sum $L \oplus T$ and report its vertices. Is $L \oplus T$ equivalent to $T \oplus L$?

5: Configuration Spaces (0.5 + 0.5)

(a.) Determine the configuration space of a system of two robots A_1 and A_2 in a two-dimensional Euclidean workspace. A_1 is a disk and A_2 is line segment. A_1 and A_2 can move freely but must always remain in contact.

(b.) Give a tight upper bound on the asymptotic complexity of the configuration space obstacle induced by a convex robot with m vertices and a convex obstacle with n vertices?

6: Kinematics for Linkages (2.0)

Consider the three-axis robot on the separate sheet. Use the first part of the Denavit-Hartenberg algorithm to assign four frames to the robot and draw the frames in the figure. Use the second part of the Denavit-Hartenberg algorithm to determine the joint angle θ_i, the joint distance d_i, the link length a_i, and the link twist angle α_i for each of the axes $i = 1, 2, 3$.

7: Form Closure Grasps (1.0)

Let R be the rectangle with corners $(0,0)$, $(10,0)$, $(10,4)$, and $(0,4)$. Let $p_1 = (5,0)$, $p_2 = (10,1)$, $p_3 = (5,4)$, and $p_4 = (0,2)$. Use Reuleaux' half-plane analysis of instantaneous velocity centers to either show that frictionless point contacts at p_1, p_2, p_3, and p_4 put R in form closure, or to identify the points that could still serve as velocity centers.

8: Force Closure Grasps (1.0)

The boundary of the convex semi-algebraic object

$$O = \{(x,y) \in \mathbb{R}^2 | x^2 + y^2 - 25 \leqslant 0\} \cap \{(x,y) \in \mathbb{R}^2 | y - 1 \leqslant 0\}$$

consists of one circular arc and one line segment. Place four frictionless point fingers along the boundary of O that put O in force closure. Use wrench analysis to show that the resulting grasp is indeed force closure.