Final Test
Motion and Manipulation

November 8, 2019
17:00-19:30

Note: It is not allowed to use pocket calculators or consult books, notes, slides, etc. Fill out your name and student number on each page you hand in. The test consists of **nine** exercises. **Motivate all your answers!**

1: Kinematics I (0.5 + 1.0)

We are given a fixed orthonormal frame $F = \{f^1, f^2, f^3\}$ and a mobile orthornormal frame $M = \{m^1, m^2, m^3\}$. Initially the frames M and F coincide. We rotate M by $\pi/2$ radians about f^3 and then by $\pi/2$ radians about f^1.

(a.) Determine the transformation matrix that maps mobile M coordinates into fixed F coordinates.

(b.) The above composition of two rotations is equivalent to a single rotation. Determine the axis and the angle of this rotation.

2: Kinematics II (0.5 + 1.0)

(a.) Compute the Hamiltonian product $(1 + 2i + 3j)(3 + 2j + k)$ and simplify the result as much as possible.

(b.) Use quaternions to determine the image of the point $p = (0, 4, 0)$ after a rotation by an angle of $\pi/2$ about the line through the origin with direction vector $(1, 0, 1)^T$.

3: Kinematics for Linkages (1.0)

Consider the four-axis robot on the separate sheet and the frames assigned to its axes of motion and its hand. Use the given frames to determine the joint angle θ_i, the joint distance d_i, the link length a_i, and the link twist angle α_i for each of the axes $i = 1, 2, 3, 4$. Clearly indicate which parameters are variable.

4: Inverse Kinematics (1.0)

We consider a two-link arm in two-dimensional Euclidean space with two rotational degrees of freedom. It consists of a long link of length 4 and a short link of length 3 that share a common endpoint that acts as a rotational joint. The other endpoint of the long link is anchored at the origin about which the link can rotate. The free endpoint of the short link is the *tip* of our arm. Joint angle θ_1 denotes the (counterclockwise) angle between the positive x-axis and the long link, and joint angle θ_2 denotes the (counterclockwise) angle between the extension of the long link and the short link.

The goal is to determine values for θ_1 and θ_2 that place the tip at $(5, 5)$. We consider the use of the Cyclic Coordinate Descent method to this inverse kinematics problem. We start from the initial configuration $\theta_1 = 0$ and $\theta_2 = 0$ in which the arm is stretched and aligns with the positive x-axis. Perform one iteration of the method through both joints (in the appropriate order). What are the coordinates of the tip after the first rotation of a joint, and what are the coordinates of the tip after the second rotation of a joint?

5: Short Questions (0.5 + 0.5)

(a.) Determine the configuration space for a system of two polyhedral entities A_1 and A_2 moving independently in a three-dimensional Euclidean workspace, where A_1 moves while one of its vertices is confined to the plane $z = 0$ and A_2 moves while one of its facets slides on the plane $z = 0$.

(b.) What is the difference between open-loop control and closed-loop control?

6: Minkowski Sums (0.5 + 0.5)

(a.) Let I be the segment with endpoints $(1, 0)$ and $(1, 4)$. Let s be the line segment with endpoints $(-2, -1)$ and $(2, 1)$ and t be the line segment with endpoints $(-2, 1)$ and $(2, -1)$. Define $X = s \cup t$. Construct the (non-convex) Minkowski sum $I \oplus X$ and list its vertices.

(b.) Let $R = \{(x, y) \mid x^2+y^2-9 \leqslant 0\} \cap \{(x, y) \mid -x^2-y^2+4 \leqslant 0\}$ and $D = \{(x, y) \mid x^2+y^2-1 \leqslant 0\}$. Construct the Minkowski sum $R \oplus D$.

7: Plücker Coordinates (1.0)

Let ℓ be the line through the points $(2, 0, -1)$ and $(-1, 1, -3)$. What is the distance between the line ℓ and the origin O?

8: Form Closure Grasps (1.0)

(a.) Let $p_1 = (0, 1)$, $p_2 = (0, 3)$, $p_3 = (-2, 3)$, $p_4 = (-2, -1)$, $p_5 = (0, -1)$, $p_6 = (0, -3)$, $p_7 = (2, -3)$, and $p_8 = (2, 1)$. Let P be the non-convex object bounded by the edges p_1p_2, p_2p_3, p_3p_4, p_4p_5, p_5p_6, p_6p_7, p_7p_8, and p_8p_1. Use Reuleaux' half-plane analysis of instantaneous velocity centers to check whether frictionless point fingers at p_1 and p_5 put P in form closure.

(b.) Let $q_1 = (1, 1)$, $q_2 = (1, 3)$, $q_3 = (-2, 3)$, $q_4 = (-2, -1)$, $q_5 = (-1, -1)$, $q_6 = (-1, -3)$, $q_7 = (2, -3)$, and $q_8 = (2, 1)$. Let Q be the non-convex object bounded by the edges q_1q_2, q_2q_3, q_3q_4, q_4q_5, q_5q_6, q_6q_7, q_7q_8, and q_8q_1. Use Reuleaux' half-plane analysis of instantaneous velocity centers to check whether frictionless point fingers at q_1 and q_5 put Q in form closure.

9: Force Closure Grasps (1.0)

We are given the square S with corners $(1, 1)$, $(-1, 1)$, $(-1, -1)$, and $(1, -1)$ and three points $p_1 = (1, 0)$, $p_2 = (0, 1)$, and $p_3 = (-1, 0)$. Consider force-closure grasps involving frictionless point fingers at p_1, p_2, p_3, and any fourth point p_4 along the boundary of S. Reason about the corresponding wrenches to prove that no such four-finger grasp can yield force closure.