Exam Pattern Recognition

9 May 2005, 15-17 hour

Remarks:

i It is not allowed to consult books, notes, telephone, etc., or someone else's answers.

ii Put your name on every sheet, and on the first sheet your student number as well.

iii **Always explain your answer, used symbols, etc.; be precise.**

iv All questions weight equal.

iv Answers may be given in Dutch or English.

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1. Statistics

   (a) What is a covariance matrix?

   (b) What is a mixed probability density function?

   Sketch of the answer:

   (a) Extent to which two variables vary together, deviate from mean. $Cov(x_i, x_j) = E((x_i - \mu_i)(x_j - \mu_j))$, where E is the expectation, and μ_i is the mean value of variable x_i.

   (b) $f(x) = \sum P_i f_i$, with $P_i = P(\omega_i)$ the a priori probability, and $f_i(x) = P(x|\omega_i)$ the class-conditional probability density.

2. Feature analysis
   What is non-parametric supervised learning? How works the k-nearest neighbor estimator?

   Sketch of the answer:
   Learning a distribution function, when model of that function is not known.

$$\hat{f}(x) = \frac{n}{N\,V}$$

   where $n = k$ and V is the volume of the smallest sphere that contains k training objects. Note: a k-nearest neighbor pdf estimator is not a k-nearest neighbor classifier!

3. Classifier
   What is a proportional classifier?

   Sketch of the answer:
   A classifier that does not always assign the same feature vector to the same class. Rather, it assigns to a class with a chance that is proportional to the probability of that feature vector. Assign to class A with probability q_A:

$$q_A = \frac{P_A f_a(x)}{P_A f_A(x) + P_B f_B(x)}$$

4. Error Analysis
   What is the Bayes error probability? Give an example for two classes A and B.

   Sketch of the answer:
   Theoretically minimal error probability.

$$\epsilon^* = \int \min\{P_A f_A(x), P_B f_B(x)\} dx$$

5. Pattern matching formulation

   (a) Give a formulation of the computation problem of geometric pattern recognition.

   (b) Give a formulation of the optimization problem of geometric pattern recognition.

   Sketch of the answer:

   (a) Compute $d(A, B)$.

   (b) Given patterns A and B, a distacne function d, and a transformation group G, compute g that minimizes d:

$$argmin_{g \in G} d(g(A), B)$$

6. Distance
   What is the triangle inequality of a distance function? Give an example of a distance function *not* satisfying this condition.

   Sketch of the answer:
   $d(x, z) \leq d(x, y) + d(y, z)$ voor alle $x, y, z \in S$.

7. Distance
   What is the Minkowski-distance between two k-dimensional points?

   Sketch of the answer:

$$L_p(x, y) = (\sum_{i=1}^{k} |x_i - y_i|^p)^{1/p}$$

8. Transformations
   What is a 2D similarity transformation, and what are its degrees of freedom?

   Sketch of the answer:

$$M = \begin{pmatrix} \epsilon s \cos\phi & -s \sin\phi & t_1 \\ \epsilon s \sin\phi & s \cos\phi & t_2 \\ 0 & 0 & 1 \end{pmatrix}.$$

   where ϵ is plus or minus one. There are four degrees of freedom: s, ϕ, t_1, and t_2.