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In 2009-2010, the course INFOSIM was given by Dr. Ir. J. M. van den Akker.

Simulation (INFOSIM)
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Answers may be provided in either Dutch or English. All your answers should be clearly written down and provide a clear explanation. Unreadable answers may be judged as false.

The home care company

We consider a home care company in the center of the Netherlands. In a certain area, it employs 10 nurses and 20 assistants. In this area, there are n customers requiring care from the company. On a typical day the care for customer i ($i = 1, 2, \ldots, n$) requires time from a nurse which equals the maximum of 10 minutes and an amount following a normal distribution with an average of p_i and a variance of v_i^g. Moreover, it requires an amount of time from an assistant which equals the maximum of 20 minutes and an amount following a normal distribution with an average of q_i and a variance of w_i^2. For different reasons (e.g. lifting the (customer) the work of the nurse and assistant have to start at *exactly the same time*. This implies that if one of the two arrives earlier, then she/he has to wait for the other before any work can start.

You may assume that travelling from one customer to another always 5 minutes. Furthermore, customers are assigned to care personnel in increasing order of their index i. This implies that, if a nurse has finished her/his work, the next customer she/he is going to serve is the one which has not been assigned to a nurse yet. The same is true for the assistants. Assume that all nurses and assistants work from 8.00 AM until approximately 5.00 PM. This means that any work started before 4.45 PM has to be finished before going home and no new service is started after 4.45 PM. Nurses as well as assistants have a 30 minutes break just after the first service finished after 12 o?clock.

Since the waiting lists are ever increasing, the company wants to serve more customers, but they are not sure if they have enough capacity for serving all these customers. Let $\{n + 1, \ldots, m\}$ with $m > n$ be the set of additional customers on the waiting list. The required service times are distributed in the same way as that of the other customers.

The company wants to perform a simulation study to find out how many customers from the additional set can be served. Note that the list of additional customers is ordered on priority of the customers, so if customer j will be served then customer $j - 1$ also has to be served because of its higher priority.

Discrete-event modeling

Question 1. (10p)

Which events are included in the event-scheduling model for this problem? Draw an event graph and for each are give the corresponding time delay.

Question 2. (5p)

What are the state variables that have to be included in the simulation?

Question 3. (7p)

Describe in words or pseudo code the event-handlers of the event(s) which concern a nurse.

Question 4. (7p)

Give two performance measures that are appropriate for this simulation study. For each of these performance measures, show how it should be computed within the simulation program.

Question 5. (10p)

It turns out that nurses prefer to work with certain assistants and vice versa. This results in a set P of preferred nurse-assistant combinations, i.e.

$$P = \{(j,k)|\text{nurse } j \text{ and assistant } k \text{ prefer to work together}\}.$$

It turns out that if customer i is served by a preferred nurse-assistant combination then the variance in the service time distribution of the nurse becomes smaller. A similar effect is observed for the service times of the assistants.

a) What will be the effect on the number of customers that can be served? Explain your answer?

b) It is clear that maximizing the number of preferred combinations used incremes the happiness of the care personnel which is very important for the organization. Describe a strategy to maximize this number and show how this strategy can be included in the simulation.

Stochastic aspects

Question 6. (8p)

The organization has an information desk where customers telephone to in case of questions. On a typical working day, telephone calls arrive according to a Poisson process with an average of 10 per hour. The duration of the calls follows an exponential distribution with an average of 5 minutes. There is one employee continuously available for answering the calls. If the employee is busy when a call is made, the customer is put into a FIFO queue. What is the average waiting time for a customer calling the desk? Explain your answer.

Question 7. (5p)

Does the simulation of questions 1-4 get into a steady state? Explain your answer.

Question 8. (8p)

Until now we assumed that the travelling time between two customers was exactly 5 minutes. Suppose that the travelling times of care personnel are subject to disturbances δ (in minutes). If a nurse or assistant travels from one customer to another, the travelling time equals $5 + \delta$ minutes, where δ is generated form a probability distribution with the following density function:

$$f_\delta(x) = \begin{array}{ll} 0.2 * 60\mathrm{e}^{60x} & \text{for } x < 0; \\ 0.8 * 30\mathrm{e}^{-30x} & \text{for } x \geq 0 \end{array}$$

Explain the meaning of this density function.
How can we generate these disturbances in a program written in an imperative programming language like Java or C++ and without using *any* specific random generation libraries or functions?
Note: You do not have to give a program, but just a description or pseudo-code.

Question 9. (20p)

The organization owns a shop where people can rent or buy specific products. For rollators, which have to be bought, the demand is normally distributed with a known expected value and variance. It is known that the average demand per four weeks is 40 with a variance of 36 per four weeks. The weekly demands are assumed to be independent. The shop wants to use the (r, q)-model as a basis for its inventory management.

a) Explain the (r, q)-model

b) Given the fixed ordering cost 100 and the holding cost 0.2 EURO per item per week, determine the optimal value of q.

c) Given a constant lead time of one week and a desired Stock-Out?Probability(SOP) = 0.025, compute the value of r. What is the size of the safety stock in this example? *A statistical table is attached.*

d) Suppose that the warehouse aims at an order fulfillment rate of 95%. Explain how this affects the values of r and q computed before (you do not have to compute the exact values).

Optimization

Question 10. (20p)

The organization wants to apply a more advanced planning tool to determine the assignment of nurses and assistants to customers and the expected starting time of the service of customer i. We consider the case without preferred nurse-assistant combinations and disturbances in travelling times. They consider to apply a tool based on an algorithm $\mathcal{A}$ that maximizes the number of customers that can be served and provides a robust planning. Robustness means trying to avoid replanning and to avoid the sitiuation that planned tasks have to be performed after working time. The algorithm $\mathcal{A}$ uses the expected values and variances of the service times as input. In case replanning is needed the algorithm $\mathcal{A}_{\mathcal{R}}$ is available. We want to evaluate the performance of the algorithm $\mathcal{A}$ by a discrete-event simulation.

a) What are the events in this simulation?

b) What is the state that has to be maintained?

c) Describe the event-handler of the event(s) in which the replanning algorithm $\mathcal{A}_{\mathcal{R}}$ is called.

TABLE T.1

Critical points $t_{\nu,\gamma}$ for the t distribution with ν df, and z_γ for the standard normal distribution

$\gamma = P(T_\nu \le t_{\nu,\gamma})$, where T_ν is a random variable having the t distribution with ν df; the last row, where $\nu = \infty$, gives the normal critical points satisfying $\gamma = P(Z \le z_\gamma)$, where Z is a standard normal random variable

	γ															
ν	0.6000	0.7000	0.8000	0.9000	0.9333	0.9500	0.9600	0.9667	0.9750	0.9800	0.9833	0.9875	0.9900	0.9917	0.9938	0.9950
1	0.325	0.727	1.376	3.078	4.702	6.314	7.916	9.524	12.706	15.895	19.043	25.452	31.821	38.342	51.334	63.657
2	0.289	0.617	1.061	1.886	2.456	2.920	3.320	3.679	4.303	4.849	5.334	6.205	6.965	7.665	8.897	9.925
3	0.277	0.584	0.978	1.638	2.045	2.353	2.605	2.823	3.182	3.482	3.738	4.177	4.541	4.864	5.408	5.841
4	0.271	0.569	0.941	1.533	1.879	2.132	2.333	2.502	2.776	2.999	3.184	3.495	3.747	3.966	4.325	4.604
5	0.267	0.559	0.920	1.476	1.790	2.015	2.191	2.337	2.571	2.757	2.910	3.163	3.365	3.538	3.818	4.032
6	0.265	0.553	0.906	1.440	1.735	1.943	2.104	2.237	2.447	2.612	2.748	2.969	3.143	3.291	3.528	3.707
7	0.263	0.549	0.896	1.415	1.698	1.895	2.046	2.170	2.365	2.517	2.640	2.841	2.998	3.130	3.341	3.499
8	0.262	0.546	0.889	1.397	1.670	1.860	2.004	2.122	2.306	2.449	2.565	2.752	2.896	3.018	3.211	3.355
9	0.261	0.543	0.883	1.383	1.650	1.833	1.973	2.086	2.262	2.398	2.508	2.685	2.821	2.936	3.116	3.250
10	0.260	0.542	0.879	1.372	1.634	1.812	1.948	2.058	2.228	2.359	2.465	2.634	2.764	2.872	3.043	3.169
11	0.260	0.540	0.876	1.363	1.621	1.796	1.928	2.036	2.201	2.328	2.430	2.593	2.718	2.822	2.985	3.106
12	0.259	0.539	0.873	1.356	1.610	1.782	1.912	2.017	2.179	2.303	2.402	2.560	2.681	2.782	2.939	3.055
13	0.259	0.538	0.870	1.350	1.601	1.771	1.899	2.002	2.160	2.282	2.379	2.533	2.650	2.748	2.900	3.012
14	0.258	0.537	0.868	1.345	1.593	1.761	1.887	1.989	2.145	2.264	2.359	2.510	2.624	2.720	2.868	2.977
15	0.258	0.536	0.866	1.341	1.587	1.753	1.878	1.978	2.131	2.249	2.342	2.490	2.602	2.696	2.841	2.947
16	0.258	0.535	0.865	1.337	1.581	1.746	1.869	1.968	2.120	2.235	2.327	2.473	2.583	2.675	2.817	2.921
17	0.257	0.534	0.863	1.333	1.576	1.740	1.862	1.960	2.110	2.224	2.315	2.458	2.567	2.657	2.796	2.898
18	0.257	0.534	0.862	1.330	1.572	1.734	1.855	1.953	2.101	2.214	2.303	2.445	2.552	2.641	2.778	2.878
19	0.257	0.533	0.861	1.328	1.568	1.729	1.850	1.946	2.093	2.205	2.293	2.433	2.539	2.627	2.762	2.861
20	0.257	0.533	0.860	1.325	1.564	1.725	1.844	1.940	2.086	2.197	2.285	2.423	2.528	2.614	2.748	2.845
21	0.257	0.532	0.859	1.323	1.561	1.721	1.840	1.935	2.080	2.189	2.277	2.414	2.518	2.603	2.735	2.831
22	0.256	0.532	0.858	1.321	1.558	1.717	1.835	1.930	2.074	2.183	2.269	2.405	2.508	2.593	2.724	2.819
23	0.256	0.532	0.858	1.319	1.556	1.714	1.832	1.926	2.069	2.177	2.263	2.398	2.500	2.584	2.713	2.807
24	0.256	0.531	0.857	1.318	1.553	1.711	1.828	1.922	2.064	2.172	2.257	2.391	2.492	2.575	2.704	2.797
25	0.256	0.531	0.856	1.316	1.551	1.708	1.825	1.918	2.060	2.167	2.251	2.385	2.485	2.568	2.695	2.787
26	0.256	0.531	0.856	1.315	1.549	1.706	1.822	1.915	2.056	2.162	2.246	2.379	2.479	2.561	2.687	2.779
27	0.256	0.531	0.855	1.314	1.547	1.703	1.819	1.912	2.052	2.158	2.242	2.373	2.473	2.554	2.680	2.771
28	0.256	0.530	0.855	1.313	1.546	1.701	1.817	1.909	2.048	2.154	2.237	2.368	2.467	2.548	2.673	2.763
29	0.256	0.530	0.854	1.311	1.544	1.699	1.814	1.906	2.045	2.150	2.233	2.364	2.462	2.543	2.667	2.756
30	0.256	0.530	0.854	1.310	1.543	1.697	1.812	1.904	2.042	2.147	2.230	2.360	2.457	2.537	2.661	2.750
40	0.255	0.529	0.851	1.303	1.532	1.684	1.796	1.886	2.021	2.123	2.203	2.329	2.423	2.501	2.619	2.704
50	0.255	0.528	0.849	1.299	1.526	1.676	1.787	1.875	2.009	2.109	2.188	2.311	2.403	2.479	2.594	2.678
75	0.254	0.527	0.846	1.293	1.517	1.665	1.775	1.861	1.992	2.090	2.167	2.287	2.377	2.450	2.562	2.643
100	0.254	0.526	0.845	1.290	1.513	1.660	1.769	1.855	1.984	2.081	2.157	2.276	2.364	2.436	2.547	2.626
∞	0.253	0.524	0.842	1.282	1.501	1.645	1.751	1.834	1.960	2.054	2.127	2.241	2.326	2.395	2.501	2.576