Utrecht University
Faculty of Science
Department of Information and Computing Sciences 2010?

Additional Exam Simulation, Monday August 23, 14.00-17.00 hr.

- Answers may be provided in either Dutch or English.
- A statistical table is attached.
- Please write down your answers clearly. Unreadable or unclear answers may be judged as false.
- All your answers should be clearly explained.
- Please write down your name and student number on every exam paper that you hand in.
- You are allowed to use a calculator.
- **Mobile phones should be switched off and put far away.**

The maximum score (100 in total) is as follows:

	Assignment 1	**Assignment 2**
a	10	12
b	8	8
c	10	8
d	10	8
e	12	14
total	50	50

Good luck, veel succes !!!

Assignment 1

At an airport, we consider check-in desks of a rather small airline. The check-in time of an individual passenger follows an exponential distribution with an average of 2 minutes. The desks are opened from 6 AM until 10 PM. The airline has the possibility to operate at most 5 check-in desks depending on the number of passengers. There is one queue for the economy class check-in desks. Passengers arrive according to a Poisson process. The intensity varies during the day and is given in the following table:

Time period	Average number of passengers per hour
6-10 AM	90
10 AM - 4 PM	70
4-7 PM	120
7-10 PM	50

At 6 AM one desk is opened. When a passenger arrives and the queue length gets equal to 10, one additional check-in desk is opened, of course unless all desks are already opened. On the other hand, when a ground steward(ess) has been idle for 15 minutes, he (she) will leave the desk unless he (she) is the only person left at the economy check-in desks. Moreover, if a steward(ess) has been working for 3 hours without breaks, he (she) can leave after finishing the currently served passenger (if any). If this results in closing the last desk or if the queue consists of ten or more persons, the steward(ess) will be replaced immediately. An arriving passenger finding more than one idle steward(ess), will randomly select one. The airline wants to perform a simulation study to evaluate their strategy of opening and closing check-in desks and to determine how many ground steward(esse)s they should employ.

(a) Which are the events that have to be included in an event-scheduling model in this simulation? Draw the event-graph corresponding to these events. For each arc include the corresponding delay with which the event is scheduled.

(b) What is the state of the system that has to be maintained during the simulation?

(c) Define three appropriate performance measures for the above simulation study. Explain how these performance measures can be computed during the simulation.

(d) Describe the event-handler in words or pseudo-code for the event(s) identified in part (a) which include the finishing of the check-in of a passenger and for the event(s) that include the decision to close a check-in desk.

In the future the number of passengers will increase; a growth of 30 percent is foreseen over the next two years. Moreover, luggage handling robots are being developed. Experiments with protoypes indicate that these robots can significantly speed up the check-in and the airline is seriously interested in using these robots.

(e) 1) How should these items be included in the simulation model? 2) What are the diffculties in validating the new simulation model? 3) What are possible ways to solve these difficulties?

Assignment 2

We consider a distribution center (DC) that maintains inventory for 100 different products. Product i ($i = 1, \ldots, 100$) has a holding cost of h_i per item per day. Orders from customers for the products arrive during office hours (8.00-17.00) according to a Poisson process with an average of 150 orders per working day. If possible, orders are delivered to customers directly from stock. If products are not available at the arrival of an order, we use backlogging, i.e., the number of items of a product that cannot be delivered is modelled as negative inventory (and will be delivered later). For product i this brings a backlogging cost b_i per item per day. The distribution center uses the following so-called (r, Q) strategy for *re-ordering* products at its suppliers. As soon as the inventory of product i gets below r_i an amount of Q_i is ordered at the supplier. A supplier order of product i has a fixed ordering cost of K_i and the delivery time of the goods is d_i working days on average. The management of the distribution center wants to determine a good re-ordering strategy. The distribution center has a fixed capacity of 1,000,000 items.

(a) It turns out that the content of a customer order has the following probability distribution. The amount of products in the order is at most 15. For $k = 1, 2, \ldots 15$, p_k denotes the probability that the order contains exactly k products. Given that there are k products in the order each $k-$tupel from the 100 product has equal probability. If product i is in the order, the ordered number of items is an integer n_i with $l_i \leq n_i \leq u_i$ such that all possible quantities have equal probability. How can we generate these order contents in a program written in an imperative programming language like Java or C++ and without using any specific libraries?
Note: You do not have to give a program, but just a description or pseudo-code.

(b) Suppose we run a simulation for a given re-ordering strategy (r_i, Q_i) ($i = 1, \ldots, 100$). Does the simulation of the above problem get into a steady state? Explain your answer.

(c) Let 2000, 1900, 2200, 2300, and 2050 be the average daily inventory cost that have been observed in 5 independent runs of a simulation of a given re-ordering strategy (r_i, Q_i). Determine $\bar{X}(5), S^2(5)$, and a 90 percent confidence-interval for the expected value μ and explain the meaning of the computed quantities.

(d) Next year, the DC will also store an additional product 101. Suppose for this product 101, the holding cost h_{101} are 3 EURO per day and the demand is fixed and equals 150 items per working day. The supplier charges an ordering cost $K_{101} = 100$. This is one the few products for which there is no delivery time. What are the values of r and Q for this product? Explain your answer?

It turns out that for a part of the products there are alternative suppliers who can deliver faster, i.e. the delivery time decreases, but charge higher ordering cost. For products $i = 1, 2 \ldots 50$, there is another supplier who charges ordering cost K_i' and has an average delivery time of d_i' working days, where $K_i' > K_i$ and $d_i' < d_i$. A given product i should always be ordered at the same supplier. So if the management of the DC decides to sign a contract with the alternative supplier for product i, it always orders product i there.

(e) Describe the above problem as a combined optimization and simulation problem, i.e. formulate (either in words or formulas) the decision variables, objective function and the constraints and indicate at which point a simulation has to be performed.

TABLE T.1

Critical points $t_{\nu,\gamma}$ for the t distribution with ν df, and z_γ for the standard normal distribution

$\gamma = P(T_\nu \le t_{\nu,\gamma})$, where T_ν is a random variable having the t distribution with ν df; the last row, where $\nu = \infty$, gives the normal critical points satisfying $\gamma = P(Z \le z_\gamma)$, where Z is a standard normal random variable

	γ															
ν	0.6000	0.7000	0.8000	0.9000	0.9333	0.9500	0.9600	0.9667	0.9750	0.9800	0.9833	0.9875	0.9900	0.9917	0.9938	0.9950
1	0.325	0.727	1.376	3.078	4.702	6.314	7.916	9.524	12.706	15.895	19.043	25.452	31.821	38.342	51.334	63.657
2	0.289	0.617	1.061	1.886	2.456	2.920	3.320	3.679	4.303	4.849	5.334	6.205	6.965	7.665	8.897	9.925
3	0.277	0.584	0.978	1.638	2.045	2.353	2.605	2.823	3.182	3.482	3.738	4.177	4.541	4.864	5.408	5.841
4	0.271	0.569	0.941	1.533	1.879	2.132	2.333	2.502	2.776	2.999	3.184	3.495	3.747	3.966	4.325	4.604
5	0.267	0.559	0.920	1.476	1.790	2.015	2.191	2.337	2.571	2.757	2.910	3.163	3.365	3.538	3.818	4.032
6	0.265	0.553	0.906	1.440	1.735	1.943	2.104	2.237	2.447	2.612	2.748	2.969	3.143	3.291	3.528	3.707
7	0.263	0.549	0.896	1.415	1.698	1.895	2.046	2.170	2.365	2.517	2.640	2.841	2.998	3.130	3.341	3.499
8	0.262	0.546	0.889	1.397	1.670	1.860	2.004	2.122	2.306	2.449	2.565	2.752	2.896	3.018	3.211	3.355
9	0.261	0.543	0.883	1.383	1.650	1.833	1.973	2.086	2.262	2.398	2.508	2.685	2.821	2.936	3.116	3.250
10	0.260	0.542	0.879	1.372	1.634	1.812	1.948	2.058	2.228	2.359	2.465	2.634	2.764	2.872	3.043	3.169
11	0.260	0.540	0.876	1.363	1.621	1.796	1.928	2.036	2.201	2.328	2.430	2.593	2.718	2.822	2.985	3.106
12	0.259	0.539	0.873	1.356	1.610	1.782	1.912	2.017	2.179	2.303	2.402	2.560	2.681	2.782	2.939	3.055
13	0.259	0.538	0.870	1.350	1.601	1.771	1.899	2.002	2.160	2.282	2.379	2.533	2.650	2.748	2.900	3.012
14	0.258	0.537	0.868	1.345	1.593	1.761	1.887	1.989	2.145	2.264	2.359	2.510	2.624	2.720	2.868	2.977
15	0.258	0.536	0.866	1.341	1.587	1.753	1.878	1.978	2.131	2.249	2.342	2.490	2.602	2.696	2.841	2.947
16	0.258	0.535	0.865	1.337	1.581	1.746	1.869	1.968	2.120	2.235	2.327	2.473	2.583	2.675	2.817	2.921
17	0.257	0.534	0.863	1.333	1.576	1.740	1.862	1.960	2.110	2.224	2.315	2.458	2.567	2.657	2.796	2.898
18	0.257	0.534	0.862	1.330	1.572	1.734	1.855	1.953	2.101	2.214	2.303	2.445	2.552	2.641	2.778	2.878
19	0.257	0.533	0.861	1.328	1.568	1.729	1.850	1.946	2.093	2.205	2.293	2.433	2.539	2.627	2.762	2.861
20	0.257	0.533	0.860	1.325	1.564	1.725	1.844	1.940	2.086	2.197	2.285	2.423	2.528	2.614	2.748	2.845
21	0.257	0.532	0.859	1.323	1.561	1.721	1.840	1.935	2.080	2.189	2.277	2.414	2.518	2.603	2.735	2.831
22	0.256	0.532	0.858	1.321	1.558	1.717	1.835	1.930	2.074	2.183	2.269	2.405	2.508	2.593	2.724	2.819
23	0.256	0.532	0.858	1.319	1.556	1.714	1.832	1.926	2.069	2.177	2.263	2.398	2.500	2.584	2.713	2.807
24	0.256	0.531	0.857	1.318	1.553	1.711	1.828	1.922	2.064	2.172	2.257	2.391	2.492	2.575	2.704	2.797
25	0.256	0.531	0.856	1.316	1.551	1.708	1.825	1.918	2.060	2.167	2.251	2.385	2.485	2.568	2.695	2.787
26	0.256	0.531	0.856	1.315	1.549	1.706	1.822	1.915	2.056	2.162	2.246	2.379	2.479	2.561	2.687	2.779
27	0.256	0.531	0.855	1.314	1.547	1.703	1.819	1.912	2.052	2.158	2.242	2.373	2.473	2.554	2.680	2.771
28	0.256	0.530	0.855	1.313	1.546	1.701	1.817	1.909	2.048	2.154	2.237	2.368	2.467	2.548	2.673	2.763
29	0.256	0.530	0.854	1.311	1.544	1.699	1.814	1.906	2.045	2.150	2.233	2.364	2.462	2.543	2.667	2.756
30	0.256	0.530	0.854	1.310	1.543	1.697	1.812	1.904	2.042	2.147	2.230	2.360	2.457	2.537	2.661	2.750
40	0.255	0.529	0.851	1.303	1.532	1.684	1.796	1.886	2.021	2.123	2.203	2.329	2.423	2.501	2.619	2.704
50	0.255	0.528	0.849	1.299	1.526	1.676	1.787	1.875	2.009	2.109	2.188	2.311	2.403	2.479	2.594	2.678
75	0.254	0.527	0.846	1.293	1.517	1.665	1.775	1.861	1.992	2.090	2.167	2.287	2.377	2.450	2.562	2.643
100	0.254	0.526	0.845	1.290	1.513	1.660	1.769	1.855	1.984	2.081	2.157	2.276	2.364	2.436	2.547	2.626
∞	0.253	0.524	0.842	1.282	1.501	1.645	1.751	1.834	1.960	2.054	2.127	2.241	2.326	2.395	2.501	2.576