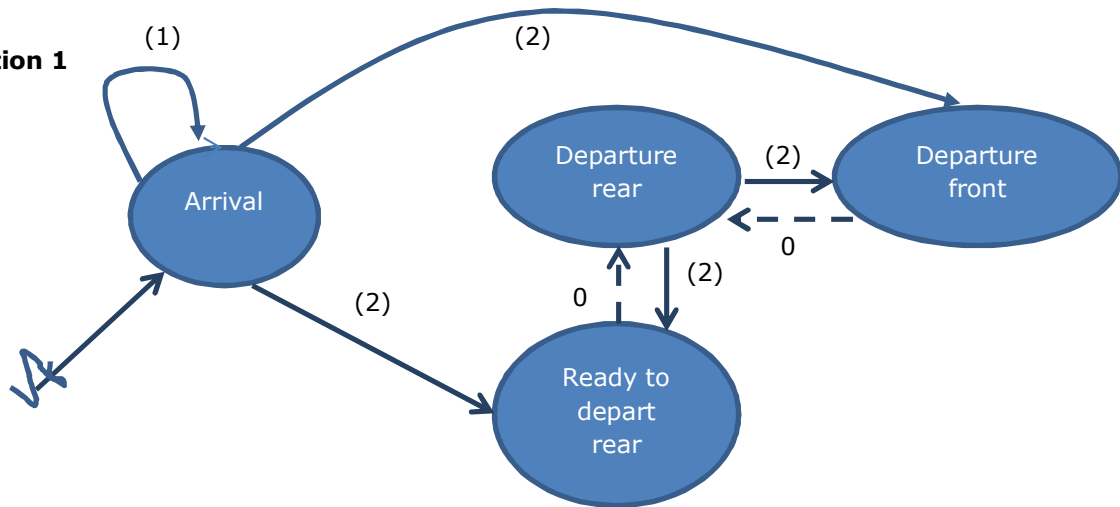


Solutions to final exam Simulation Thu April 9, 2015

Question 1



- (1) $A_i^{\text{scheduled}} + N(0,3) - A_{i-1}^{\text{actual}}$ (note that we made the assumption: $A_{i-1}^{\text{actual}} < A_i^{\text{actual}}$, if you do not make this assumption you have to generate A_i^{actual} before the start of the simulation and sort them in increasing order)
- (2) $\text{Max}(U([1;6]), d_i^{\text{scheduled}} - \text{now})$

NB: Departure front does not have to schedule itself, since it is impossible that there is a queue of waiting buses and an empty rear position at the same time.

Question 2

Average waiting time in the queue: when you schedule 'Ready to depart' or 'Departure front' for a bus, you know the waiting time W_i of the bus in the queue (may be 0). You maintain $W = \sum_i W_i$ and at the end of the simulation you divide W by the number of buses.

Utilization factor of the platforms: for each platform you have to maintain the time during which it has been occupied by a bus, say B_i is the total busy time of platform i . You update this at every event by adding $\text{current_time} - \text{time_previous event}$ to B_i to all platforms that have been occupied by a bus since that last event. An alternative way is to update B_i in the event 'Departure rear' and 'Departure front' when these events concern platform i .

Question 3

- Use your simulation model to find the average departure delay for the current number of platforms and check if this coincides with the real average departure delay in your current model.
- Check if buses always go to the front position when it is free, and do not stop at the rear position on this case.
- Check if buses who are waiting in the queue do not disembark passengers at another position than the platform.

Question 4

$$z_i = x_i(9 \text{ platforms}) - x_i(10 \text{ platforms})$$

$$z \text{ - values : } 1, 2, -1, 0, 1, 0, 2, 1, 1, 0$$

$$\bar{z} = 0.7$$

$$S^2(10) = \frac{\sum_{i=1}^{10} (z_i - \bar{z})^2}{9} = 0.9$$

$$t_{n-1, 1-\alpha/2} = 2.262$$

$$95\% \text{ confidence interval : } 0.7 \pm 2.262 \sqrt{\frac{0.9}{10}} = 0.7 \pm 0.679,$$

$$\text{hence } [0.021; 1.379]$$

All number in the confidence interval are positive. Therefore, the average departure delay is decreased if we go from 9 to 10 platforms.

Question 5

(1) We have a composite distribution, so we first have to determine whether the deviation is positive or negative. We generate U_1 from $U[0,1]$ by a linear congruential generator ($Z_{i+1} = a Z_i \text{ mod } m$, where m is a large prime number e.g. $2^{31}-1$).

(2) If $U_1 \leq 0.1$, then generate U_2 from $U[0,1]$ and set deviation $D = -0.5 U_2$

(3) Otherwise you have to use inverse transform

$$u = F(x) = 1 - e^{-\frac{x}{2}} \Leftrightarrow e^{-\frac{x}{2}} = 1 - u \Leftrightarrow x = -2 \ln(1 - u)$$

Hence generate U_3 from $U[0,1]$ and set deviation $D = -2 \ln(1 - U_3)$

Question 6

- In the (r, q) model r is the reorder point, which means that you order additional supplies if your inventory goes below r items. q is the replenishment quantity, i.e., if you order, you always order an amount of q items.
- EOQ formula (make sure that you take both demand and inventory cost per week)

$$q^* = \sqrt{\frac{2KE(D)}{h}} = \sqrt{\frac{2 * 196 * 1000}{0.2}} = 1400$$

$$SOP = \frac{hq^*}{c_B E(D)} = \frac{0.2 * 1400}{11.2 * 1000} = 0.025$$

The weekly demands are independent and normally distributed. So the weekly demand follow a normal distribution with an average of 1000 and variance of 90 (standard deviation 9.48) So we must have

$$P(d_{week} \geq r) = 0.025$$

$$P\left(\frac{d_{week} - 1000}{9.48} \geq \frac{r^* - 1000}{9.48}\right) = 0.025$$

⇒ (statistical table standard normal distribution)

$$\frac{r^* - 1000}{9.48} = 1.96 \Leftrightarrow r^* = 1018.77$$

The safety stock is r^* - demand during the lead time, hence $9.48 * 1.96 = 18.77$.

Question 7

There is a decision variable for each arriving bus trip, which indicates the platform to which that bus trip is assigned.

The objective function is the average departure delays of the buses.

To find the value of the objective function, a simulation has to be performed.

An example of a solution algorithm is simulated annealing:

1. Determine starting solution $x(start)$ and starting temperature $T(start)$, set $k=0$
2. Set current solution $x = x(start)$
3. If $k = k(temp-decrease)$, decrease T to aT and set $k=0$, otherwise $k=k+1$,
4. Determine new solution $neighbour(x)$

5. Set $x = neighbour(x)$ with probability

$$P(\text{accept } neighbour(x)) = \begin{cases} 1 & \text{if } R(neighbour(x)) \leq R(x) \\ e^{-\frac{R(neighbour(x)) - R(x^{(n)})}{T}} & \text{otherwise} \end{cases}$$

6. If x is better than the best solution so far x_{best} ,

set $x_{best} = x$

7. Go to Step3, unless stopcriterium is met

You can determine a neighbor by randomly generating a bus and changing the platform of this bus to the platform which currently serves the least number of busses