DEPARTMENT OF PHYSICS AND ASTRONOMY, FACULTY OF SCIENCE, UU.
MADE AVAILABLE IN ELECTRONIC FORM BY THE $\mathcal{T}_{\!\mathcal{B}}\mathcal{C}$ OF A–ESKWADRAAT
IN 2010-2011, THE COURSE NS-101B WAS GIVEN BY .

Speciale Relativiteitstheorie (NS-101b)
12 november 2010

- The exam consists of three exercises, all of which count for 30%.
- This exame counts for 90% of the final mark (the homework exam for 10%)

Formularium
In this exam, we will always assume inertial observers O and O' with synchronized clocks. O' has a constant speed v, relative to O.

- The special Lorentz transformations are

$$x' = \gamma(x - vt) \; ; \qquad t' = \gamma(t - \frac{v}{c^2})x, \tag{1}$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \; , \qquad \beta \equiv \frac{v}{c} \tag{2}$$

- The energy and momentum of a particle with mass m and speed v are given by $E = mc^2\gamma$ and $p = mv\gamma$. For a massless particle, we have the relation $E = pc$.

Question 1. Doppler's law from the Lorentz transformations

Use the special Lorentz transformations to derive the formula for the relativistic Doppler effect,

$$f' = \frac{f}{k(\beta)} \; , \qquad k(\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}} \; , \tag{3}$$

where f is the frequency of the light sent out by the source O, and f' is the frequency measured by the observer O' moving relative to the source with constant speed $v = \beta c$. The direction of the speed of O' is the same as the direction of propagation of the light. To derive Doppler's law, you may go through the following steps:

a) Let the source O emit a light signal to O' at every time step $t = T, 2T, \ldots,$ with frequency $f = 1/T$. Draw the spacetime diagram of O and indicate the events of emission and reception as points in the diagram.

b) Determine the spacetime coordinates of the receiving events in the frame of O, in terms of T, v, and the speed of light c.

c) Lorentz transform these coordinates to the frame of O' and determine from this the frequency f'. Show that your result reproduces Doppler's law (3).

Question 2. A moving rod

A rod is directed along the x-axis and moves along this direction with constant speed v, relative to an observer O. The rest-length of the rod is $2L_0$, as measured in the rod's restframe O'. At $t = 0$, the midpoint of the rod is located at $x = 0$. Now consider a circular ring of (rest-)radius L_0 which, in the frame of O, moves with constant speed along the z-axis. The ring is always parallel to the (x, y)-plane and at $t = 0$ the center of the ring is at the origin in the (x, y)-plane at $z = 0$.

a) What is the length of the rod as measured in the frame of O? Draw a picture of the rod and the ring in the (x, y)-plane at $t = 0$. Does the rod fit into the ring?

b) Determine the time(s) at which the ring is crossing the x'-axis according to the observer in the restframe O'.

c) Draw a picture of the situation of the rod and the ring, as seen from along the z'-axis, paying attention to the Lorentz contraction that O' measures. Describe what happens as seen by an observer in the rest-frame O'.

Question 3. Pion decay

A neutral pion moves in the laboratory along the x-axis and decays into two photons (lightparticles). The energy E of the pion is twice its rest-energy E_0, with $E_0 = 135$ MeV (Mega-electronVolt).

a) What is the speed of the pion, relative to the speed of light?

b) Compute the energy of the two photons, assuming that they are emitted along the x-axis in opposite directions.

[Hint: $\sqrt{3} \approx 1,73$.]