FACULTEIT NATUUR- EN STERRENKUNDE
UNIVERSITEIT UTRECHT

FINAL EXAM Special Relativity 2010

1) Start every exercise on a separate sheet. Write on each sheet: your name and initials, and your studentnumber.

2) Please write legibly and clear. Unreadable handwriting cannot be marked!

3) The exam consists of **three** exercises, all of which count for 30%.

4) No lectures notes or any other material (books, calculators, ...) are allowed. A formularium instead is given.

Formularium

In this exam, we will always assume inertial observers O and O' with synchronized clocks. O' has a constant speed v, relative to O.

- The special Lorentz transformations are

$$x' = \gamma(x - vt) \; ; \qquad t' = \gamma(t - \frac{v}{c^2}x) \; , \tag{1}$$

  where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \; , \qquad \beta \equiv \frac{v}{c} \; . \tag{2}$$

- The energy and momentum of a particle with mass m and speed v are given by $E = mc^2\gamma$ and $p = mv\gamma$. For a massless particle, we have the relation $E = pc$.

1. Doppler's law from the Lorentz transformations

Use the special Lorentz transformations to derive the formula for the relativistic Doppler effect,

$$f' = \frac{f}{k(\beta)} \; , \qquad k(\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}} \; , \tag{3}$$

where f is the frequency of the light sent out by the source O, and f' is the frequency measured by the observer O' moving relative to the source with constant speed $v = \beta c$. The direction of the speed of O' is the same as the direction of propagation of the light. To derive Doppler's law, you may go through the following steps:

a) Let the source O emit a light signal to O' at every time step $t = T, 2T, \ldots$, with frequency $f = 1/T$. Draw the spacetime diagram of O and indicate the events of emission and reception as points in the diagram.

b) Determine the spacetime coordinates of the receiving events in the frame of O, in terms of T, v, and the speed of light c.

c) Lorentz transform these coordinates to the frame of O' and determine from this the frequency f'. Show that your result reproduces Doppler's law (3).

2. A moving rod

A rod is directed along the x-axis and moves along this direction with constant speed v, relative to an observer O. The rest-length of the rod is $2L_0$, as measured in the rod's restframe O'. At $t = 0$, the midpoint of the rod is located at $x = 0$. Now consider a circular ring of (rest-)radius L_0 which, in the frame of O, moves with constant speed along the z-axis. The ring is always parallel to the (x, y)-plane and at $t = 0$ the center of the ring is at the origin in the (x, y)-plane at $z = 0$.

a) What is the length of the rod as measured in the frame of O ? Draw a picture of the rod and the ring in the (x, y)-plane at $t = 0$. Does the rod fit into the ring ?

b) Determine the time(s) at which the ring is crossing the x'-axis according to the observer in the restframe O'.

c) Draw a picture of the situation of the rod and the ring, as seen from along the z'-axis, paying attention to the Lorentz contraction that O' measures. Describe what happens as seen by an observer in the rest-frame O'.

3. Pion decay

A neutral pion moves in the laboratory along the x-axis and decays into two photons (lightparticles). The energy E of the pion is twice its rest-energy E_0, with $E_0 = 135$ MeV (Mega-electronVolt).

a) What is the speed of the pion, relative to the speed of light ?

b) Compute the energy of the two photons, assuming that they are emitted along the x-axis in opposite directions.

[Hint: $\sqrt{3} \approx 1,73$.]