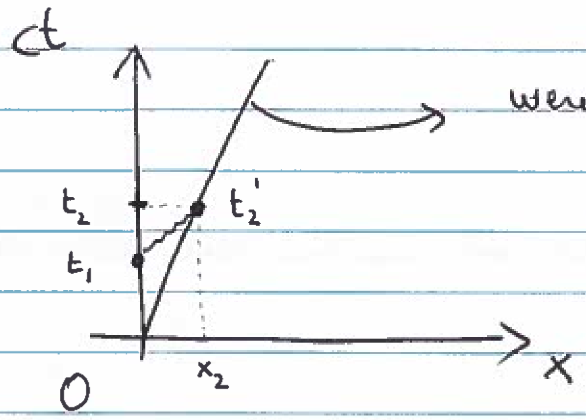
Oplossing vraag 1

1) Ruimtetijd diagram



2) Uit het diagram volgt

$$v = \frac{x_2}{t_2} \quad \text{en} \quad c = \frac{x_2}{t_2 - t_1}$$

$$\Rightarrow \quad x_2 = v\,t_2 \quad \text{en} \quad c = \frac{v\,t_2}{t_2 - t_1} \quad \Rightarrow \quad t_2 = \frac{t_1}{1 - v/c}$$

3) Gebruik de Lorentz-transformatie

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$ toegepast op gebeurtenis (t_2, x_2):

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = \gamma\left(t_2 - \frac{v^2}{c^2}t_2\right) = t_2\,\gamma\left(1 - \frac{v^2}{c^2}\right)$$

$$= t_2\sqrt{1 - \frac{v^2}{c^2}} = t_1\sqrt{\frac{1 - v^2/c^2}{(1 - v/c)^2}} = t_1\sqrt{\frac{1 + v/c}{1 - v/c}}$$

$\boxtimes$

Oplossing vraag 2

1) Voor constante kracht $\vec{F}$ volgt uit $\vec{F} = \frac{d\vec{p}}{dt}$ dat

$$\vec{p}(t) = \vec{F}t + \vec{p}_0 \quad \text{en} \quad \vec{p} = m\gamma_u \vec{u}$$

Componentsgewijs :

- $p_x = p_0$ en $p_x = m\gamma_u u_x \Rightarrow \boxed{\gamma_u u_x = \frac{p_0}{m}}$

- $p_y = Ft$ en $p_y = m\gamma_u u_y \Rightarrow \boxed{\gamma_u u_y = \frac{Ft}{m}}$

2) Energie $E = m\gamma_u c^2$, en bereken γ_u.

Uit 1) volgt $\gamma_u^2 (u_x^2 + u_y^2) = \frac{p_0^2 + (Ft)^2}{m^2}$

$$\Rightarrow \frac{1}{1 - u^2/c^2} \cdot \frac{u^2}{c^2} = \frac{p_0^2 + (Ft)^2}{m^2c^2}$$

$$\Rightarrow \frac{u^2}{c^2}\left[1 + \frac{p_0^2 + (Ft)^2}{m^2c^2}\right] = \frac{p_0^2 + (Ft)^2}{m^2c^2}$$

$$\Rightarrow \frac{u^2}{c^2} = \frac{p_0^2 + (Ft)^2}{p_0^2 + (Ft)^2 + m^2c^2} \qquad (< 1)$$

$$\Rightarrow \gamma_u^{-2} = 1 - \frac{u^2}{c^2} = \frac{m^2c^2}{p_0^2 + (Ft)^2 + m^2c^2}$$

$$\Rightarrow E(t) = m\gamma_u c^2 = \frac{\cancel{m} c^{\cancel{2}}}{\cancel{m}\cancel{c}} \sqrt{p_0^2 + (Ft)^2 + m^2c^2} = \underline{\underline{\sqrt{E_0^2 + (Fct)^2}}}$$

3) Uit 1) en 2) volgt

$$u_x = \gamma_u^{-1} \frac{p_0}{m} = \frac{p_0 c}{\sqrt{p_0^2 + m^2c^2 + (Ft)^2}} \xrightarrow{t \to \infty} 0$$

$$u_y = \gamma_u^{-1} \frac{Ft}{m} = \frac{Fct}{\sqrt{p_0^2 + m^2c^2 + (Ft)^2}} \xrightarrow{t \to \infty} c$$

$$u^2 = u_x^2 + u_y^2 = \frac{(p_0^2 + (Ft)^2) \cdot c^2}{p_0^2 + m^2c^2 + (Ft)^2} \leqslant c^2$$

Oplossing vraag 3

Energie en impulsbehoud :

$$E_H = E_{\gamma_1} + E_{\gamma_2} \qquad \gamma_{1,2} : \text{foton 1 en 2}$$

$$\vec{p}_H = \vec{p}_{\gamma_1} + \vec{p}_{\gamma_2}$$

1) Ruststelsel Higgs : $\vec{p}_H = 0$

$$\Rightarrow \vec{p}_{\gamma_1} = -\vec{p}_{\gamma_2} \Rightarrow |p_{\gamma_1}| = |p_{\gamma_2}| \equiv |p_\gamma|$$

Verder is $E_H = E_H^\circ = m_H c^2$ en $E_\gamma = |p_\gamma| c$

$$\Rightarrow m_H c^2 = 2 |p_\gamma| c$$

$$\Rightarrow |p_\gamma| = \frac{m_H c}{2} \quad \text{en} \quad E_\gamma = \frac{m_H c^2}{2}$$

2) Laboratorium stelsel. Stel energie van een foton (foton "1") wordt gemeten en genoteerd met E_γ. We lossen dan op voor foton "2" :

$$E_{\gamma_2} = E_H - E_\gamma = \sqrt{\vec{p}_{\gamma_2}^{\,2} \cdot c^2} = |\vec{p}_{\gamma_2}| c$$

$$\vec{p}_{\gamma_2} = \vec{p}_H - \vec{p}_{\gamma_1}$$

We rekenen uit: $(\vec{p}_{\gamma 2} = \vec{p}_2)$ $\qquad E_H^2 = p_H^2 c^2 + m_H^2 c^4$

$$\vec{p}_2 \cdot \vec{p}_2 = p_H^2 + p_1^2 - 2\, \vec{p}_H \cdot \vec{p}_1$$

$$= \frac{1}{c^2}\left(E_H^2 - m_H^2 c^4\right) + \frac{E_\gamma^2}{c^2} - 2\,|p_H|\,|p_1| \cos\theta$$

We hebben ook $\vec{p}_2 \cdot \vec{p}_2 = \frac{E_{\gamma 2}^2}{c^2} = \frac{(E_H - E_\gamma)^2}{c^2}$

$$\Rightarrow \cancel{\frac{E_H^2}{c^2}} + \cancel{\frac{E_\gamma^2}{c^2}} - \frac{2\, E_H E_\gamma}{c^2} = \cancel{\frac{E_H^2}{c^2}} + \cancel{\frac{E_\gamma^2}{c^2}} - m_H^2 c^2$$

$$- 2\,|p_H| \frac{E_\gamma}{c} \cos\theta$$

$$\Rightarrow \frac{2\, E_H E_\gamma - m_H^2 c^4}{c^2} = 2\,|p_H| \frac{E_\gamma}{c} \cos\theta$$

$\Rightarrow$

$$\cos\theta = \frac{2\, E_H E_\gamma - m_H^2 c^4}{2\, E_\gamma c\, |p_H|}$$

$$= \frac{2\, E_H E_\gamma - m_H^2 c^4}{2 E_\gamma \sqrt{E_H^2 - m_H^2 c^4}}$$

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