Relativiteitstheorie (NS-101b)
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Opgave 1: Een deeltje in een lab

a) Als de kinetische energie van een deeltje met massa m in het lab precies N keer zo groot is als de rustenergie van het deeltje, wat is dan de snelheid van het deeltje?

b) Als dit deeltje instabiel is en rust een verwachte levensduur heeft van 11 minuten, wat is dan de waargenomen verwachte levensduur?

c) Kan dit deeltje, voor het vervalt, een foton uitzenden met een frequentie ω, gemeten in het lab, zodat het in het lab tot stilstand komt? Zo ja; geef een uitdrukking voor ω in termen van m en N. Zo nee; waarom niet?
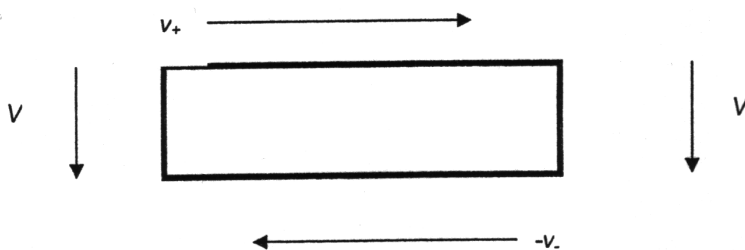
Opgave 2: Pionverval

Een pion met een massa m kan vervallen in een muon met massa μ en een massaloos neutrino.

a) Bekijk dit verval in het ruststelsel van het pion. Geef een uitdrukking voor de energie van het muon in termen van de massa's van het muon en het pion.

b) Als het pion voor het verval in het lab langs de x-as bewoog met een snelheid van $0.5c$. Geef de impuls van het muon in het lab.

Opgave 3: Verborgen impuls

Beschouw de volgende rechthoek gemaakt van geleidend draad waardoor elektronen met een massa m stromen. De rechthoek heeft een lange zijde met lengte L en een korte zijde met lengte B. Langs de korte zijdes doorlopen de elektronen een constant potentiaal verschil V waardoor ze versnellen, dan wel afremmen. Daardoor hebben de elektronen onder een snelheid v_- en boven in de geleider een snelheid v_+.

Als N_+ en N_- het aantal elektronen is dat boven en onder in de lange zijde stromen dan volgt uit het feit dat de stroom overal gelijk is dat $N_+ v_+ = N_- v_-$.

a) Bereken de totale impuls die door de rechthoek stroomt (daarbij mag je aan nemen dat de bijdragen van de korte zijdes tegen elkaar wegvallen). Doe dit relativistisch en niet-relativistisch.

b) De stroom l in de kring is gelijk aan

$$I = \frac{eN_+ v_+}{L} = \frac{eN_- v_-}{L}$$

De energie winst die er op de rechter korte zijde geboekt wordt is dus IV. Druk de totale relativistische impuls in de kring uit in de stroom l en het potentiaal verschil V.

Wat je hebt uitgerekend is de zogenaamde "verborgen impuls" in het systeem die weg valt tegen de impuls die in de elektrische en magnetische velden is gaan zitten.

Opgave 4: Bonus: Ehrenfest paradox

Lees het artikel van T.A. Weber uit Am.J.Phys. en leid af hoe hij aan vergelijking (2) komt.

note on rotating coordinates in relativity

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In this paper, the rotating disk as an example of a non-Euclidean space is carefully examined; the basic arguments of Einstein are emphasized. A new approach is also presented which resolves the Ehrenfest paradox. © *1997 American Association of Physics Teachers.*

Einstein, in his popular book,[1] *The Meaning of Relativity,* es a rotating coordinate system as an example of a non-lidean space. We reconstruct his arguments while em-sizing the principles involved. We also present a new roach which gives the same result and shows that a ma-al body set into rotational motion experiences stretching gent to circles centered on the axis of rotation.

Consider a region where there are no effects of gravity. K be an inertial frame of reference and let the frame $\bar{K}$ te with constant angular velocity ω about a fixed point O K. For both frames choose cylindrical coordinates with mon origin at O and with the z and $\bar{z}$ axes aligned along axis of rotation. We restrict r by $\omega r < c$ so that the co-inate system of $\bar{K}$ can be realized by markings on a ma-al body. Here, c is the velocity of light in vacuum and ωr he velocity of a point in $\bar{K}$ as observed by K. In K let C a circle of radius r centered at O and perpendicular to the s rotation. Let $\bar{C}$ be the locus of points in $\bar{K}$ that coincide C at any given time. $\bar{C}$ will be a circle of radius $\bar{r}$. We w that the circumference of the circle C divided by its us is 2π since we have Euclidean geometry in inertial nes of reference. Einstein showed that the ratio, $\bar{R}$, of the umference of $\bar{C}$ as measured in $\bar{K}$ to its radius is greater n 2π. Such a measurement presents a problem since it olves an interpretation of coordinates in a non-inertial ref-nce frame.

We can start with the invariant interval as described by the rdinates in K:

$$ds^2 = c^2 \, dt^2 - dr^2 - r^2 \, d\phi^2 - dz^2, \tag{1}$$

ere the time is measured by a set of synchronized clocks ced at convenient fixed positions in K and cylindrical co-inates are used for the spatial part. Arbitrary transforma-s to new coordinates will leave the interval unchanged; y the space-time description of events will be changed. w all of these frames of reference with new space-time rdinates are equally valid. Einstein was led to this con-sion by thoughtful consideration of the local equivalence an accelerated frame to a gravitational field. Even though these frames are equally valid we know the form of the s of nature only in inertial frames where distance is mea-

sured with standard rods and time intervals by standard clocks. Even in the case of a space time manifold including gravitational fields,[2] a transformation can be made to a local inertial coordinate system such that the geodesic equation is a straight line, that is, coordinates in which particles behave as if ''free'' of gravitational forces or inertial forces. Einstein refers to observers in such coordinates as freely falling. From the known laws of physics in the inertial system we can then infer the form of the laws in the general system.

There is no need to make any detailed algebraic transfor-mations to apply the above procedure to the measurement of the circumference and the radius of $\bar{C}$ (see Adler, Bazin, and Schiffer[3] for the details of such a transformation). The mea-surement of the length of any short segment in the plane of $\bar{C}$ can be done in an inertial frame that is instantaneously at rest with respect to the segment. Imagine using many short stan-dard rods placed end to end along a curve to measure its length. Do this along the circumference of $\bar{C}$, with each rod in the instantaneous rest system of the segment it is to mea-sure. As observed simultaneously from K, the rods appear Lorentz contracted and lie end to end along the circle $\bar{C}$ which is superimposed on C. Since the rods are shortened as compared to a rod at rest in K, the number along the circum-ference of $\bar{C}$ is greater by the factor $(1 - \omega^2 r^2/c^2)^{-1/2}$ than the number of rods in K needed to measure the circumfer-ence of C. Do the same for the radius and find that $r = \bar{r}$ since lengths perpendicular to the motion are not Lorentz contracted. We therefore conclude that the circumference of $\bar{C}$, as measured by freely falling standard rods instanta-neously at rest with respect to the segments of $\bar{C}$, divided by its radius is

$$\bar{R} = 2\pi (1 - \omega^2 r^2/c^2)^{-1/2}. \tag{2}$$

At this point one might ask about a reciprocal measure-ment of C as observed from $\bar{K}$. It might be mistakenly thought that the measured circumference of C would be larger than $\bar{C}$. Certainly such reciprocity exists in the mea-surements of lengths between two inertial frames, but for a rotating system coordinate clocks cannot be synchronized throughout space.[2] Without synchronization of clocks, con-

for the construction of these equally spaced radial lines
either K or $\bar{K}$. For definiteness take the number to be 360.
In each frame these radial lines divide the circumference of
the corresponding circle into 360 equal intervals. As observed simultaneously from K, the length of the intervals on
the circle $\bar{C}$ must appear to be exactly the same length as the
intervals on the circle C. If they appeared otherwise, a count
of the intervals on the circle $\bar{C}$ would not be 360. This is
possible since from K one can observe all the radial lines
of $\bar{K}$ simultaneously. Note that this applies for any angular
velocity of the rotating frame, even during an acceleration of
the frame from rest to its final angular velocity. For the intervals to appear the same length in spite of the Lorentz
contraction, the actual rest length of an interval in $\bar{K}$ must be
greater by the factor $(1-\omega^2 r^2/c^2)^{-1/2}$. This confirms our earlier result and leads to an interesting conclusion. Suppose the
coordinate system is realized by markings on a material disk.
Then the material within the intervals must physically stretch
as the disk, starting at rest, is brought up to rotational speed
ω.

Notice that the above arguments, based solely on relativistic kinematics, offers a resolution of the Ehrenfest
paradox.[4-6] The paradox is as follows: As observed from the
inertial system K, the Lorentz contraction acts only on the
periphery and not on the radius of the disk. It is therefore
supposed that the ratio of the circumference to the radius for
the rotating disk is less than 2π, an apparent violation of the
Euclidean geometry of an inertial frame! But, as seen above,
an increase in rest length compensates for the Lorentz contraction. This is consistent with the statement of Ø. Grøn[7]
that "relativistic kinematics alone forbids giving the disk a
motion so that the rest lengths of the elements of the periphery remain constant during the period of angular acceleration." In detail, consider two closely spaced marks on the
periphery of the disk which is initially at rest. As observed
from K, the two marks have exactly the same angular acceleration as the disk is brought up to rotational speed ω. It
follows from simple kinematics that the distance between the

$$\Delta t = t_0[(1-\omega^2 r^2/c^2)^{-1/2} - 1], \tag{3}$$

where l_0 is the unstretched length of the spring. As stated
earlier, the material of the disks within the intervals must
physically stretch. This change in rest length is entirely different than the change in size associated with observations
made between moving inertial frames in special relativity.
Here it is assumed that the disk will elastically deform rather
than fragment.

Discussion of the forces involved and the elastic properties of the material of the disk are beyond the scope of the
present note. Some dynamical aspects of the rotating disk
can be found in the papers by Clark,[8] Cavaller,[9] Brotus,[10]
and McCrea.[11]

ACKNOWLEDGMENT

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SIMPLICITY VS. UGLINESS

Ever since 't Hooft's 1971 paper I had been quite convinced of the correctness of the outlines
of this theory, but I regarded the particular version of this theory that Salam and I had constructed
as only one specially simple possibility. For instance, there might be other members of the family
formed by the photon and the W and Z particles, or other particles related to the electron and
neutrino. Pierre Duhem and W. Van Quine pointed out long ago that a scientific theory can never
be absolutely ruled out by experimental data because there is always some way of manipulating
the theory or the auxiliary assumptions to create an agreement between theory and experiment. At
some point one simply has to decide whether the elaborations that are needed to avoid conflict
with experiment are just too ugly to believe.

Steven Weinberg, *Dreams of a Final Theory* (Pantheon Books, New York, 1992), p. 125.

Formuleblad

Consider a frame S' moving with a speed v with respect to a frame S;

$$\beta = \frac{v}{c} \tag{1}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

Time-dilation and Length contraction

$$t = \gamma(v) t_0 \tag{3}$$

$$l = \frac{1}{\gamma(v)} l_0 \tag{4}$$

Lorentz transformations

$$x' = \gamma(v)(x - \beta ct) \tag{5}$$

$$y' = y \tag{6}$$

$$z' = z \tag{7}$$

$$ct' = \gamma(v)(ct - \beta x) \tag{8}$$

Kinematics

$$v_1' = \frac{v_1 - v}{1 - \frac{v_1 v}{c^2}} \tag{9}$$

Energy and Momentum

$$E = m\gamma(v)c^2 \tag{10}$$

$$\vec{p} = m\gamma(v)\vec{v} \tag{11}$$

If $m = 0$,

$$E = |\vec{p}|c \tag{12}$$

Relativistic energy of photons according to De Broglie

$$E = \hbar\omega \tag{13}$$

$$|\vec{p}| = \frac{\hbar 2\pi}{\lambda}$$

$$= \hbar|\vec{k}| \tag{14}$$

Doppler shift

$$\omega' = \sqrt{\frac{c+v}{c-v}}\omega \tag{15}$$

if S' approaches the source head-on. **Forces**

$$\vec{f_\perp} = m\gamma(v)\vec{a_\perp} \tag{16}$$

$$\vec{f_{//}} = m\gamma(v)\vec{a_{//}} \tag{17}$$

Spacetime vectors
Position-vector of an event X

$$\mathbf{X} = ct\mathbf{e_0} + x\mathbf{e_1} + y\mathbf{e_2} + z\mathbf{e_3} \tag{18}$$

in S'

$$\mathbf{X} = ct'\mathbf{e_0'} + x'\mathbf{e_1'} + y'\mathbf{e_2'} + z'\mathbf{e_3'} \tag{19}$$

Innerproduct

$$\mathbf{e_i} \cdot \mathbf{e_i} \;=\; -1, \; i = 1, 2, 3 \tag{20}$$

$$\mathbf{e_i} \cdot \mathbf{e_j} \;=\; 0, i \neq j \tag{21}$$

$$\mathbf{e_i} \cdot \mathbf{e_0} \;=\; 0, \; i = 1, 2, 3 \tag{22}$$

$$\mathbf{e_0} \cdot \mathbf{e_0} \;=\; 1 \tag{23}$$