(25)

1a Ising model : magnetisatie
of Gas-vloeistof : lattice gas (5)

b $\Omega(U,N) = \dfrac{N!}{n!\,(N-n)!} = \dfrac{N!}{\frac{U}{\varepsilon}!\,\left(N-\frac{U}{\varepsilon}\right)!}$ (2)

$U = n\varepsilon$

$S = k_B \ln \Omega = N \ln N - \frac{U}{\varepsilon}\ln\frac{U}{\varepsilon} - \left(N-\frac{U}{\varepsilon}\right)\ln\left(N-\frac{U}{\varepsilon}\right)$ (2)

S maximaal bij $n = \frac{N}{2} \Rightarrow U = \varepsilon\frac{N}{2}$ (1)

minimaal bij $n=0$ en $n=N \Rightarrow U = 0, \varepsilon N$

c $\frac{1}{T} \overset{(3)}{=} \left(\frac{\partial S}{\partial U}\right) \overset{(2)}{=} -\frac{1}{\varepsilon}\ln\frac{U}{\varepsilon} + \frac{1}{\varepsilon}\ln\left(N-\frac{U}{\varepsilon}\right)$

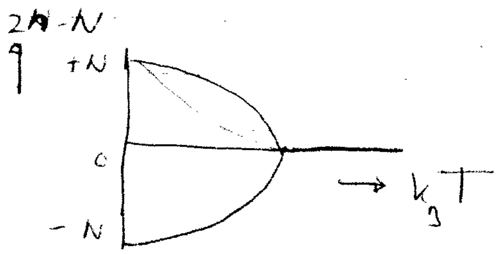
d $P_0 = \dfrac{1}{1+e^{-\beta\varepsilon}}$ (1)

$P_+ = \dfrac{e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$ (1)

$Z_1 \overset{(1)}{=} 1+e^{-\beta\varepsilon}$ $\quad Z_N \overset{(2)}{=} Z_1^{\,N} = \left(1+e^{-\beta\varepsilon}\right)^N$

e $k_B T \gg J$ even veel deeltjes met energie $-J$ als met energie $+J$ (2)

$k_B J \ll J$ fasenscheiding in fasen met veel $-J$ en een fase met veel $+J$



Grondtoestand $n=0$ of $n=N$
Hoge T $n = \frac{1}{2}N$ (3)

(25)

(2) a $Z_{1,\text{quant}} = \sum_{n=0}^{\infty} e^{-\beta\hbar\omega(n+\frac{1}{2})}$ (3)

$$= \frac{e^{-\frac{\beta}{2}\hbar\omega}}{1-e^{-\beta\hbar\omega}}$$

$$P_n = \frac{e^{-\beta\hbar\omega(n+\frac{1}{2})}}{e^{-\frac{\beta}{2}\hbar\omega}}\left(1-e^{-\beta\hbar\omega}\right) = e^{-\beta n\hbar\omega}\left(1-e^{-\beta\hbar\omega}\right)$$ (2)

b $U = -\frac{\partial \ln Z}{\partial \beta}$ (3)

$$= \frac{\hbar\omega}{2} + \frac{\hbar\omega e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}$$ (2)

c hoge T $\quad U \to \frac{\hbar\omega}{2} + k_B T \sim k_B T$ equipartitie (5)

lage T $\quad U \to \frac{\hbar\omega}{2}$ alles in grondtoestand

d $Z_{1,\text{klas}}(T) = \frac{1}{h} \int dx \int dp_x\, e^{-\left(\frac{p_x^2}{2m k_B T} + \frac{\frac{1}{2}m\omega^2 x^2}{k_B T}\right)}$

$$= \frac{1}{h}\sqrt{2\pi m k_B T}\sqrt{\frac{2\pi k_B T}{m\omega^2}}$$ (3)

$$= \frac{k_B T}{\hbar\omega}$$

hoge T $\quad Z_{1,\text{quant}} = \frac{k_B T}{\hbar\omega}$ (2)

e $U = -\frac{\partial \ln Z}{\partial \beta} = k_B T$ 2 kwadratische vrijheidsgraden (2)

$2 \cdot \frac{1}{2} k_B T = k_B T$

$C_V = \left(\frac{\partial U}{\partial T}\right)_V = k_B$ (1)

$S = \frac{U-F}{T} = k_B + \frac{k_B T \log \beta\hbar\omega}{T} = k_B + k_B \log\left(\frac{k_B T}{\hbar\omega}\right)$ (2)

3a $Z = \left(\frac{V_{cell}\, e^{\beta\varepsilon}}{\Lambda^3}\right)^N$

$F = -k_B T \log Z$

$= N k_B T \log\left(\frac{\Lambda^3}{V_f\, e^{\beta\varepsilon}}\right)$ (2)

$\mu = k_B T \log\left(\frac{\Lambda^3}{V_f\, e^{\beta\varepsilon}}\right)$ (3)

b $T_g = T_x$

$\mu_g = \mu_x$ (5)

$p_g = p_x$

c $\mu_g \overset{(2)}{=} k_B T \log(\rho_g \Lambda^3) \overset{(3)}{=} k_B T \log\left(\frac{p_g \Lambda^3}{k_B T}\right)$

d $\mu_x = k_B T \log\left(\frac{\Lambda^3}{V_f\, e^{\beta\varepsilon}}\right)$

$\mu_g = \mu_x \Rightarrow \frac{p_g}{k_B T} = \frac{1}{V_f\, e^{\beta\varepsilon}}$ (5)

~~d~~ e b = volume van atoom (1)

$= 1 - 10\ \text{Å}^3$

Kritiek punt: $\frac{\partial p}{\partial V} = 0$

$\frac{\partial^2 p}{\partial V^2} = 0$

$V_k^* = 3b$

$k_B T^* = \frac{8a}{27 N b}$ (3)

$T > T^*$ geen onder-scheid tussen gas en vloeistof (1)

$T < T^*$ gas-vloeistof. continu comprimeerbaar, druk neemt monotoon toe met dichtheid

4a $m \frac{\partial \vec{v}(t)}{dt} = 0$ voor $t \to \infty$ (2)

$\overline{F} = 6\pi \eta a \vec{v}_d (t \to \infty)$

$\vec{v}_d (t \to \infty) = \frac{\overline{F}}{6\pi \eta a}$ (3)

b $\frac{m\, d\vec{v}(t)}{dt} = -6\pi \eta a \vec{v}(t)$

$\vec{v}(t) = \vec{v}_0 \, e^{-\frac{6\pi\eta a}{m} t}$ (5)

c $\langle x^2(t) \rangle = \int \frac{x^2}{\sqrt{4\pi D t}} e^{-\frac{x^2}{4Dt}} dx$

$= 2Dt$ (5)

d $a^2 = 2Dt_d$ (2)

$t_d = \frac{a^2}{2D} = \frac{a^2 \, 6\pi\eta a}{2 k_B T} = \frac{3\pi}{2} \frac{a^3}{k_B T} \eta \approx 25$ [s] (3)

e $nA \rightleftarrows A_n$

$P_v \propto N_A$ ideaal gas wet (2)

$P_n \propto N_A - nN_p + N_p = N_A + (1-n) N_p$ (2)

$\frac{P_v}{P_n} = \frac{N_A}{N_A + (1-n) N_p}$ (1)