1) $$Z_1 \overset{3}{=} \sum_{\alpha} e^{-\beta E_\alpha} = \sum_{\alpha=0,1} e^{-\beta\cdot\alpha\varepsilon} \overset{3}{=} 1 + e^{-\beta\varepsilon}$$

$$\beta = 1/kT$$

$$Z(N,T) = \sum_{\alpha_1=0,1} \sum_{\alpha_2=0,1} \dots \sum_{\alpha_N=0,1} e^{-\beta(\alpha_1+\alpha_2+\dots\alpha_N)\varepsilon}$$

$$\overset{3}{=} Z_1^N \overset{!}{=} \left(1+e^{-\beta\varepsilon}\right)^N$$

2) $$\langle E\rangle \overset{!}{=} \sum_\alpha E_\alpha P_\alpha = -\frac{\partial \ln Z(N,T)}{\partial\beta} = -\frac{N}{Z_1}\frac{\partial Z_1}{\partial\beta}$$

$$\overset{!}{=} \frac{N\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}} = u$$

$$C_V \overset{!}{=} \left(\frac{\partial u}{\partial T}\right)_V = \left(\frac{\partial u}{\partial\beta}\right)\left(\frac{\partial\beta}{\partial T}\right) = -\frac{N}{kT^2}\left(-\frac{\varepsilon^2 e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}} + \frac{\varepsilon^2 e^{-2\beta\varepsilon}}{(1+e^{-\beta\varepsilon})^2}\right)$$

$$\overset{!}{=} \frac{N}{kT^2}\left(\frac{\varepsilon^2 e^{-\beta\varepsilon}}{(1+e^{-\beta\varepsilon})^2}\right)$$

$$S \overset{!}{=} \frac{u-F}{T} \overset{!}{=} N\left(\frac{\varepsilon e^{-\beta\varepsilon}}{T(1+e^{-\beta\varepsilon})} + k_B \ln\left(1+e^{-\beta\varepsilon}\right)\right)$$

Alle extensieve grootheden met λ schalen

$$N \to \lambda N$$

$$\langle E_\lambda\rangle = \frac{\lambda N\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}} = \lambda\langle E\rangle$$ extensief

$$\langle C_{v,\lambda}\rangle = \frac{\lambda N}{kT^2}\left(\frac{\varepsilon^2 e^{-\beta\varepsilon}}{(1+e^{-\beta\varepsilon})^2}\right) = \lambda\langle C_v\rangle$$ extensief 1

$$S_\lambda = \lambda N\left(\frac{\varepsilon e^{-\beta\varepsilon}}{T(1+e^{-\beta\varepsilon})} + k_B \ln\left(1+e^{-\beta\varepsilon}\right)\right) = \lambda S$$ extensief

Hoge temp. limiet

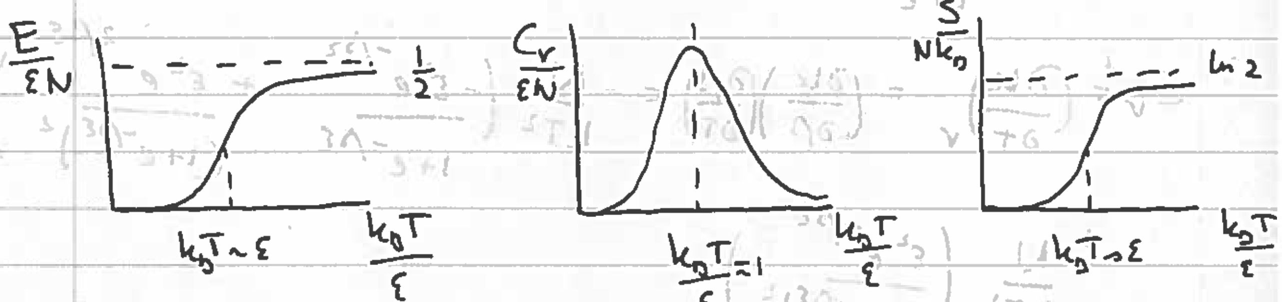
$$k_BT \gg \varepsilon \; ; \; \frac{\varepsilon}{k_BT} \to 0 \; ; \; e^{-\frac{\varepsilon}{kT}} \to 1$$

$$E \to \frac{N\varepsilon}{2} \quad C_V \to 0 \quad S \to Nk_B \ln 2$$

Lage temp. limiet

$$k_BT \ll \varepsilon \; ; \; \frac{\varepsilon}{k_BT} \to \infty \; ; \; e^{-\frac{\varepsilon}{k_BT}} \to 0$$

$$E \to 0 \quad C_V \to 0 \quad S \to 0$$



1 1 1

3) $$\sigma^2 = \langle E^2\rangle - \langle E\rangle^2 \overset{1}{=} -\frac{\partial\langle E\rangle}{\partial\beta} = -\frac{\partial}{\partial\beta}\left(-\frac{1}{Z}\frac{\partial Z}{\partial\beta}\right)$$

$$\overset{2}{=} -\frac{1}{Z^2}\left(\frac{\partial Z}{\partial\beta}\right)^2 + \frac{1}{Z}\frac{\partial^2 Z}{\partial\beta^2}$$

$$-\frac{\partial\langle E\rangle}{\partial\beta} = -\frac{\partial \frac{N\cdot\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}}{\partial\beta} = \frac{N\varepsilon^2 e^{-\beta}}{1+e^{-\beta\varepsilon}} - \frac{N\varepsilon^2 e^{-2\beta\varepsilon}}{(1+e^{-\beta\varepsilon})^2}$$

$$\overset{4}{=} \frac{N\varepsilon^2 e^{-\beta\varepsilon}}{(1+e^{-\beta\varepsilon})^2} = \sigma^2$$

$$\frac{\sqrt{\sigma^2}}{\langle N\rangle} \overset{3}{\propto} \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

4) Deeltjes zijn ononderscheidbaar, want identiek 3 (geen rugnummers) en kunnen vrij bewegen 3. Ze kunnen dus elkaars plaats innemen. N! manieren om N deeltjes op N posities te zetten: zijn dezelfde microtoestanden 2. Om voor overtelling te corrigeren delen door N! 2

5) $$Z_1 = \frac{1}{h^3}\int_V d\vec{r}_1 \int_{-\infty}^{\infty} d\vec{p}_1\, e^{-\frac{\vec{p}_1^{\,2}}{2m k_B T}}$$

$$= \frac{1}{h^3} V \cdot (2\pi m k_B T)^{3/2} \overset{5}{=} \frac{V}{\Lambda^3} \quad ; \quad \Lambda = \frac{h}{\sqrt{2\pi m k_B T}}$$

Thermische golflengte

$$Z_N = \frac{1}{N!}\frac{V^N}{\Lambda^{3N}} \overset{5}{=} \frac{1}{N!} Z_1^N$$

6) $$F \overset{3}{=} -kT \ln Z_N = -kT \ln\left[\frac{V^N}{N!\,\Lambda^{3N}}\right]$$

$$= -kT \ln V^N + kT \ln \Lambda^{3N} + kT \ln N!$$

$$\overset{\text{Stirling}}{\underset{2}{=}} NkT\left[\log\left(\frac{N}{V}\Lambda^3\right) - 1\right] \; 2$$

$$F(\lambda N, \lambda V, \lambda T) = \lambda N kT\left[\log\left(\frac{\lambda N}{\lambda V}\Lambda^3\right) - 1\right]$$

$$\overset{3}{=} \lambda F(N,V,T) \quad \text{extensief}$$

7) $$p \overset{5}{=} -\left(\frac{\partial F}{\partial V}\right)_{N,T} = -\frac{\partial NkT\left[\ln\left(\frac{N}{V}\Lambda^3\right)-1\right]}{\partial V}$$

$$= -\frac{\partial NkT \ln \frac{1}{V}}{\partial V} \overset{5}{=} \frac{NkT}{V}$$ is id. gas wet

8) $$\mu \overset{2}{=} \left(\frac{\partial F}{\partial N}\right)_{V,T} = \frac{\partial NkT\left[\ln\left(\frac{N}{V}\Lambda^3\right)-1\right]}{\partial N} \overset{2}{=} kT \ln\left(\frac{N}{V}\Lambda^3\right)$$

id. gas wet $$\mu \overset{3}{=} kT \ln\left(\frac{p}{kT}\Lambda^3\right) = kT \ln\left(\frac{p}{kT}\frac{h^3}{(\sqrt{2\pi m k})^3\, T^{3/2}}\right)$$

$$B \overset{3}{=} \frac{h^3}{k(2\pi m k)^{3/2}}$$

9) $Z_1(\mu, T) \overset{5}{=} \sum_{\alpha=0,1} e^{\beta\mu N_\alpha - \beta E_\alpha}$

$$= e^{\beta\mu\cdot 0 - \beta\cdot 0} + e^{\beta\mu\cdot 1 - \beta\varepsilon}$$

$$\overset{5}{=} 1 + e^{\beta(\mu-\varepsilon)}$$

10) $P_{geb} \overset{10}{=} \dfrac{e^{\beta(\mu-\varepsilon)}}{1+e^{\beta(\mu-\varepsilon)}}$