Modelantwoord vraag 1 Stat. fys 4-11-2015

1 a) $W(\vec{v}) \overset{!}{=} C \; e^{-\beta \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2)}$ (Boltzmann)

met $\int d\vec{v}\, W(\vec{v}) = 1$ dus $C = \dfrac{1}{\int d\vec{v}\, e^{-\beta \frac{1}{2} m \vec{v}^2}}$

$$\Rightarrow C = \frac{1}{\left(\int_{-\infty}^{\infty} dv_x\, e^{-\frac{1}{2}\beta m v_x^2}\right)^3} = \left(\frac{1}{\sqrt{2\pi k_B T/m}}\right)^3$$

$$= \left(\frac{m}{2\pi k_B T}\right)^{3/2}$$

max 3

$\langle \frac{1}{2} m \vec{v}^2 \rangle = 3 \langle \frac{1}{2} m v_x^2 \rangle = 3 \cdot \frac{1}{2} k_B T$ (equipartitie)

of berekenen $\langle \frac{1}{2} m \vec{v}^2 \rangle \overset{!}{=} \int d\vec{v}\, W(\vec{v})\, \frac{1}{2} m \vec{v}^2 = \dots = \frac{3}{2} k_B T$

max 2

b) $\langle v_x^2 \rangle - \langle v_x \rangle^2 = \langle v_x^2 \rangle = \dfrac{k_B T}{m}$

dus de standaard deviatie $\sim \sqrt{\langle v_x^2 \rangle - \langle v_x \rangle^2} = \sqrt{\dfrac{k_B T}{m}} \sim 10^2 - 10^3$ m/s

De typische afstand $= \left(\dfrac{V}{N}\right)^{1/3} \underset{\uparrow}{\simeq} \left(\dfrac{k_B T}{p}\right)^{1/3} = \left(\dfrac{4 \cdot 10^{-21}}{10^5}\right)^{1/3}$ m

want N_2 is vrijwel ideaal

$\approx (40)^{1/3} \cdot 10^{-9}$ m $= 3-4$ nm

c) $f = 3$ voor puntdeeltjes

$f \neq 3$ voor deeltjes met interne vrijheidsgraden zoals vibraties en rotaties

$$C_p = \left(\frac{dQ}{dT}\right)_p = \left(\frac{dU + p\,dV}{dT}\right)_p = \frac{f}{2} \cdot N k_B + p \frac{d}{dT}\left(\frac{N k_B T}{p}\right)_p$$

$$= \left(\frac{f}{2} + 1\right) N k_B$$

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d) $đQ = dU + pdV$ met $dU = \frac{f}{2} N k_B \, dT = 0$

want isotherm

dus $đQ = p\,dV = \frac{Nk_BT}{V} dV$ want reversibel

$$\Rightarrow Q = Nk_BT \int_V^{5V} dV \frac{1}{V} = Nk_BT \ln 5$$

e) Adiabatisch $đQ = dU + p\,dV \stackrel{2}{=} 0 = \frac{f}{2}Nk_B \, dT + \frac{Nk_BT}{V}dV$

$$\Rightarrow \frac{f}{2}\frac{dT}{T} + \frac{dV}{V} = 0 = d\left(\ln\left(T^{f/2}V\right)\right)$$

$\Rightarrow TV^{2/f}$ is $\stackrel{2}{\text{constant}}$ dus $T_e V_e^{2/f} = T_b V_b^{2/f}$

$$\Rightarrow \left(\frac{T_e}{T_b}\right) = \left(\frac{V_b}{V_e}\right)^{2/f} = \left(\frac{1}{\alpha}\right)^{2/f} \quad \left(= \alpha^{-2/f}\right)$$

Modelantwoord vraag 3 stat. fys 4-11-2015

3 a) $dG = -SdT + Vdp + \mu dN$ 5 (teken fout : max 3, (-2); term fout : (-2))

óf afleiding beginnend met $dU = TdS - pdV + \mu dN$ (1e Hfdwet) 2

geeft $d(\underbrace{U + pV - SdT}_{\equiv G}) = dU + pdV + Vdp - SdT - TdS$ 1 1

$= dG = -SdT + Vdp + \mu dN$ 1

b) $\left(\frac{\partial p}{\partial T}\right)_V \overset{1}{=} -\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial V}\right)_T \overset{1}{=} -\frac{\partial}{\partial V}\left(\frac{\partial F}{\partial T}\right)_V \overset{1}{=} \left(\frac{\partial S}{\partial V}\right)_T$ Maxwell (=3)

$\overset{2}{=} -\left(\frac{\partial S}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_S$ reciprocity

c) $T \overset{1}{=} \left(\frac{\partial U}{\partial S}\right)_V = c\,\frac{3}{2}\,(S/V)^{1/2} \Rightarrow (S/V)^{1/2} = 2T/3c$ 1

$p \underset{1}{=} -\left(\frac{\partial U}{\partial V}\right)_S = c\,\frac{1}{2}\,(S/V)^{3/2} = c\,\frac{1}{2}\left(\frac{2T}{3c}\right)^3 = \frac{4}{27c^2}\,T^3$ 1 1

d) Machine is reversibel en werkt alleen tussen T_k en T_h, en heeft dus dezelfde efficientie als een Carnot machine 3 (want elke reversibele machine heeft dezelfde efficientie bij dezelfde T's).

Dus $W/Q_h = 1 - \frac{T_k}{T_h}$ 2

e) Carnot machine dus $\frac{dQ_1}{T_1} = \frac{dQ_2}{T_2}$

Dus $\frac{-C_1\, dT_1}{T_1} = \frac{C_2\, dT_2}{T_2}$ met C_1 en C_2 de warmte caps van heet en koud bad, resp.

Dus $C_1\, d\ln T_1 + C_2\, d\ln T_2 = 0$

$$d\ln T_1 + \frac{C_2}{C_1}\, d\ln T_2 = 0$$

$$d(\ln T_1) + 2\, d\ln T_2 = 0$$

$$T_1 T_2^2 = \text{constant} = T_e^3 = T_h T_k^2$$

$$\Rightarrow T_e = T_h^{1/3}\, T_k^{2/3}$$