UTRECHT UNIVERSITY DEPARTMENT OF PHYSICS

Midterm exam in Electrodynamics

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Midterm exam
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DURATION: 3 hours
TOTAL: 115 points

1. *Use a separate sheet for every exercise.*
2. *Write your name and initials as well as your student ID number on all sheets.*
3. *A formula sheet is provided with the exam, you may not use any other formula sheets or books.*
4. *Write clearly, illegible work cannot be corrected.*
5. *Distribute your time wisely between the questions.*

Question 1. 20 points

a) Give a heuristic argument to why all the extrema of the solutions to Laplace's equation occur on the boundaries of the considered region.
(*Hint: You may assume the 'averaging property' shown in the lecture.*) (3 points)

b) State the first uniqueness theorem for the solutions to Laplace's equation. Prove this theorem. (7 points)

c) Explain the separation of variables method for solving Laplace's equation. When is it sufficient to look for solutions of this form? (5 points)

d) Is there a simple configuration of point charges with a vanishing total charge, yet non-vanishing electric field? How would you calculate the electric field of such a configuration? (5 points)

Question 2. 20 points

We observe a sphere of the radius R with the total charge Q and charge density

$$\rho(r) = \begin{cases} \alpha & \text{for } r \leq R/2 \\ 2\alpha\left(1 - \frac{r}{R}\right) & \text{for } R/2 \leq r \leq R \\ 0 & \text{for } r \geq R \end{cases},$$

where $\alpha > 0$ is a constant.

a) Determine α in terms of Q and R. (6 points)

b) Calculate the electric field inside and outside the sphere. (14 points)

Question 3. 20 points

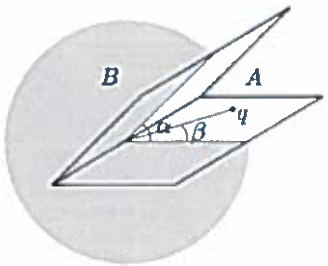
An infinite grounded conducting sheet is folded to form an angle of $\alpha = 45°$ in the xy-plane and we introduce a point charge in this region, see figure. The charge is at the unit distance from the origin and at an angle β with respect to the x-axis.

a) Calculate the potential in the region A using the method of images. (9 points)

b) Is the result from (a) valid for region B as well? Why/why not? Does the calculation from (a) work if the sheet is folded to form an angle of $\alpha = 50°$? Why/why not? (2 points)

c) What is the force on the charge q? (4 points)

d) What is the work required to bring q from infinity to the given position? (5 points)



Question 4. 15 points

A box is assembled of square metal plates with sides a, 5 of which are grounded. The top one is made of a separate sheet of metal, insulated from the others, and held at a constant potential V_0. Find the potential inside the box.

Question 5. 25 points

The potential at the surface of a sphere with radius R is given by

$$V(R, \theta, \phi) = V_0 \sin^4 \theta.$$

Find the potential inside and outside the sphere. Assume that there is no charge inside or outside the sphere.

Bonus: Question 6. 15 points

Show, in general, the following equality:

$$\frac{1}{\mathcal{R}} = \frac{1}{r} \sum_{n=0}^{\infty} \left(\frac{r'}{r} \right)^n P_n(\cos \alpha),$$

where α is the angle between $\boldsymbol{r}$ and $\boldsymbol{r}'$, and $\boldsymbol{\mathcal{R}} = \boldsymbol{r} - \boldsymbol{r}'$.