Tentamen Voortgezette Mechanica

NS-350B, Blok 2, Midterm, 14 December 2017

Mark on *each* sheet clearly your **name** and **collegekaartnummer**.
Please use a **separate sheet** for each problem.
Tip: Read all questions and start with the one you find the easiest. Do not use too much time on any one question!

1 Double Pendulum

A mathematical pendulum of length l_1 and mass m_1 is attached in point O. A second pendulum of length l_2 and mass m_2 is attached at the end of the first pendulum. All connections move without friction and may be considered massless. [total: 15 points]

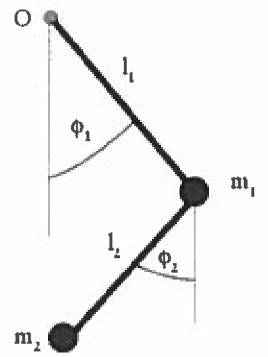

Figure 1: Double pendulum. Note that in the above situation, ϕ_2 is negative.

(a) In order to find the Lagrangian $\mathcal{L}(\phi_1, \phi_2, \dot{\phi}_1, \dot{\phi}_2, t)$ that describes this system, find the kinetic energy T and potential energy U.

Show that our choice of ϕ_1 and ϕ_2 satisfies the requirements that a Lagrangian has for its generalized coordinates. Include a short explanation of the requirements. (5+1 points)

(b) Find the generalized force and generalized momentum of each coordinate, and use them to write down the equation of motion for ϕ_1 and ϕ_2. Work carefully – the expressions are complex and have multiple terms! (3+2 points)

(c) Consider the case that $m_1 \gg m_2$ and and $l_1 \sim l_2$; first, without calculation, describe what the motion of $\phi_1(t)$ should look like and then find the solution in the limit of small oscillations. (2+2 points)

2 Projectile and Incline

A bullet of mass m is launched with initial speed v_o at an angle θ to the horizontal. You may neglect air resistance encountered by the projectile. The force of gravity acts vertically down ($a_g = -g\hat{y}$). Eventually the bullet lands on an incline which is at an angle α; upon impact, the projectile loses its velocity component normal to the incline and begins to slide up the incline. The friction encountered by the projectile on the incline is equal to μ times the normal force. [total: 10 points]

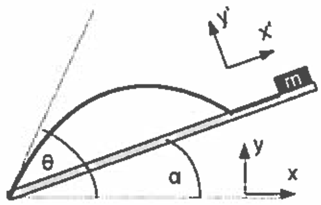

Figure 2: Mass on incline with friction.

(a) Determine all forces (magnitudes and directions) acting on the bullet before it lands on the incline and use Newton's laws to calculate how far up the incline (measured along the x' direction) the bullet lands. Find the velocity $\vec{v}_i$ of the bullet at the time of impact. *Hint:* as the bullet lands when $y' = 0$, consider using the coordinate system (x', y') for this calculation. (5 points)

(b) Determine all forces (magnitudes and directions) acting on the bullet after it lands on the incline and calculate how far up the incline (measured in the y direction) the bullet will travel from its point of impact. Check your result by comparing it to the case where there is no friction ($\mu = 0$, energy conservation). (3 points)

(c) How large does α have to be so that the bullet will slide back down the incline after it has come to a stop? (2 points)

3 Multiple Choice

The final questions for this exam are multiple choice (on a separate sheet). Please make sure to remove the staple *cleanly* before you hand in the answer sheet. [total: 5 points]

Useful Formulas

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \qquad \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\arcsin(\gamma)) = \sqrt{1-\gamma^2} \qquad \cos(\pi+\alpha) = -\cos\alpha \qquad \sin(\pi+\beta) = -\sin\beta$$

$$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta)) \qquad \sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$$

$$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$$

$$\sin^2\alpha + \cos^2\alpha = -e^{i\pi} = 1, \qquad T_0 \approx 2\pi\sqrt{\frac{\ell}{g}}$$

$$T + U + \int_S \vec{F}\cdot\vec{r}\,d\vec{r} = \text{const, where } S \text{ is the path taken.}$$

$$c = \frac{\ell^2}{\gamma\mu} \quad E = \frac{\gamma^2\mu}{2\ell^2}(\epsilon^2-1) \quad \mu = \frac{m_1 m_2}{m_1+m_2} \quad \frac{c_1}{1+\epsilon_1\cos(\phi+\delta_1)} = \frac{c_2}{1+\epsilon_2\cos(\phi+\delta_2)}$$

$$\left(\frac{d\mathbf{Q}}{dt}\right)_{S_0} = \dot{\mathbf{Q}} + \mathbf{\Omega}\times\mathbf{Q} \qquad \left(\frac{d^2\mathbf{Q}}{dt^2}\right)_{S_0} = \ddot{\mathbf{Q}} + 2\mathbf{\Omega}\times\dot{\mathbf{Q}} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{Q})$$

$$f_{\text{cstr},j}(q_i) = 0 \qquad \mathcal{H} = \sum_i p_i\dot{q}_i - \mathcal{L}$$

$$\mathcal{L}(q_i,\dot{q}_i,t) = T - V \qquad \frac{\partial\mathcal{L}}{\partial q_i} + \sum_j \lambda_j \frac{\partial f_{\text{cstr},j}}{\partial q_i} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_i}$$

$$U_{\text{cf}}(r) = \frac{\ell^2}{2\mu r^2} \qquad \mathcal{L}_{\text{rel}} = \tfrac{1}{2}\mu\dot{\mathbf{r}}^2 - U(r) = \tfrac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\phi}^2\right) - U(r)$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \mathcal{O}(x^2) \qquad T_{\text{cyl}} = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 \qquad w(\phi) = u(\phi) - C$$

$$f(a) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n; \quad f^{(n)} = \frac{d^n f}{dx^n}$$

$$I_{\text{cylinder}} = \frac{1}{2}MR^2$$