MID-TERM EXAM GEOPHYSICAL FLUID DYNAMICS
4 November 2009, 9.00 - 11.00 hours
Two problems (all items have equal weight)

Remark 1: answers may be written in English or Dutch.

Remark 2: in all questions you may use $g = 10\ \mathrm{ms}^{-2}$ and $\Omega = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}$.

Problem 1

Consider the continuity equation and momentum equations for a molecular viscous fluid on a rotating Earth, as are given on the supplementary equation sheet.

a. Specify the name of parameter f_*. Also, give its formula and explain the physical meaning of all terms in the equations that include this parameter.

b. Indicate which term(s) include the effect of the centrifugal force that is caused by the spinning of the Earth around its axis. Motivate your answer.

c. Describe the Boussinesq approximation and use it to derive a reduced version of the vertical momentum balance.

d. Assuming a constant density, derive the vertical momentum balance that results from application of the Reynolds averaging procedure.
Also, identify and parameterise the Reynolds stresses.

e. Explain the physical meaning of variable p that appears in the result of item d. Limit your answer to a few sentences.

f. What is the physical meaning of potential temperature in a dry atmosphere? Limit your answer to a few sentences.

For problem 2: P.T.O.

Problem 2

An anticyclonic circular vortex in the ocean moves from area 1 (with depth $h = h_1 = 3$ km) to a ridge where the depth is $h = h_2 = 2$ km. In both areas the tangential velocity profile in the interior of the vortex is given by

$$v_\theta = \begin{cases} -U\,\frac{r}{R} & \text{if } r \leq R, \\ 0 & \text{if } r > R. \end{cases}$$

Here, r, θ are polar coordinates. The corresponding Cartesian velocity components are $u = -\sin\theta\, v_\theta$ and $v = \cos\theta\, v_\theta$.

In area 1 the radius $R = R_1 = 50$ km and $U = U_1 = 1\ \mathrm{ms}^{-1}$. Assume the density to be constant, and $|f| = 10^{-4}\ \mathrm{s}^{-1}$, $\nu_E = 10^{-2}\ \mathrm{m}^2\mathrm{s}^{-1}$.

a. On which hemisphere is the vortex located?
   Explain your answer.

b. Compute the Rossby number of the vortex in area 1.

c. Use the geostrophic balance to derive an expression for the sea surface $\eta(r)$. Assume the sea surface elevation to be zero at the boundary of the vortex.

d. Compute the (relative) circulation of the vortex in area 1 at its boundary.

e. Sketch the velocity distribution of the vortex in the bottom Ekman layer.
   Explain your answer.

f. Derive expressions for velocity $U = U_2$ and radius $R = R_2$ when the vortex is above the ridge. Also, compute the numerical values of U_2 and R_2.

END

GFD 2009 Equation sheet

Continuity and momentum equations: molecular viscous fluid

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho\, u) + \frac{\partial}{\partial y}(\rho\, v) + \frac{\partial}{\partial z}(\rho\, w) = 0$$

$$\begin{aligned}
\rho\left(\frac{du}{dt} + f_* w - f\, v\right) &= -\frac{\partial p}{\partial x} && + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\
\rho\left(\frac{dv}{dt} + f\, u\right) &= -\frac{\partial p}{\partial y} && + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \\
\rho\left(\frac{dw}{dt} - f_* u\right) &= -\frac{\partial p}{\partial z} - \rho g && + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}$$

Energy budget for adiabatic flow of fixed composition

$$\rho\, C_v \frac{dT}{dt} - \frac{T}{\rho}\left(\frac{\partial p}{\partial T}\right)_\rho \frac{d\rho}{dt} = 0$$

Relative circulation and relative vorticity

$$\Gamma = \oint_C \mathbf{u} \cdot d\mathbf{r} = \int_S (\nabla \times \mathbf{u}) \cdot \mathbf{n}\, dS$$

where S is the surface enclosed by contour C.

Shallow water equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - f\, v &= -g\frac{\partial \eta}{\partial x} \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f\, u &= -g\frac{\partial \eta}{\partial y} \\
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) &= 0
\end{aligned}$$

Ekman pump (Northern Hemisphere)

$$\overline{w} = \frac{d}{2}\overline{\zeta}\,, \qquad d = \left(\frac{2\nu_E}{f}\right)^{1/2} \qquad \text{and} \qquad w_{\mathrm{E}k} = \frac{1}{\rho_0\, f}\left[\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right]$$