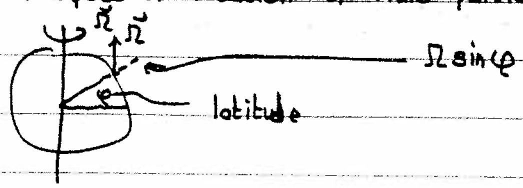
answers GFD exam d.d. 10-11-2010

1a. Coriolis parameter / planetary vorticity : $f = 2\Omega \sin\varphi$ Ω angular speed of rot. φ

$2\times$ angular speed of rotation of fluid particle around local z-axis:



1b. Richardson number $Ri = \dfrac{\Delta\rho\, g H}{\frac{1}{2}\rho_0 U^2}$

where $\Delta\rho$ density difference between top-bottom, H mean thickness, g acceleration due to gravity, ρ_0 mean density, U velocity scale.

Meaning: ratio of potential energie (due to stratification) and kinetic energy.

In this case: $\Delta\rho = 0$ (equation is that of homogeneous shallow water model)

So $Ri = 0$. So indeed, very small.

c. Pressure gradient force per mass unit in the y-direction.

To be derived from $-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}$ (see equation sheet).

Shallow water $\rightarrow$ <u>hydrostatic balance</u>, so

$$\frac{\partial p}{\partial z} = -\rho_0 g \quad \Rightarrow \quad p = \rho_0 g(\eta - z) + p_0$$

using boundary condition $p = p_0$ at $z = \eta$.

Assuming p_0 = constant: $\dfrac{\partial p}{\partial x} = \rho_0 g \dfrac{\partial \eta}{\partial x}$, so $-\dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x} = -g\dfrac{\partial \eta}{\partial x}$

Remark: you can't say $p = \rho_0 g \eta$. This only applies at level $z=0$. Here, use of hydr. balance is essential.

d. Homogeneous shallow water model: u, v ~~is~~ are independent of z, w varies linearly with z.

So $v = 1\ ms^{-1}$ at $z = b$ and $w = u\dfrac{\partial b}{\partial x} + v\dfrac{\partial b}{\partial y}$ at $z = b$

or $w = v\dfrac{\partial b}{\partial y} = 0.02\ ms^{-1}$

e. $v = 1\ ms^{-1}$ at $z = \eta$ (depth uniform).

$$w = w_{bot} + (w_{surf} - w_{bot})\frac{(z-b)}{h}$$

We know $w_{1/2} = w_{bot} + \frac{1}{2}(w_{surf} - w_{bot}) = \frac{1}{2}w_{bot} + \frac{1}{2}w_{surf}$

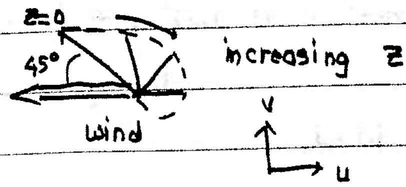
So $w_{surf} = 2w_{1/2} - w_{bot} = 0.08\ ms^{-1}$

\#

2a. At $y=0$ $\quad \tau^x = -\tau_0 + \hat{\tau} < 0$

No geostrophic flow in interior ; Northern Hemisphere

So



note : flow speed decays exponentially on scale $d = \left(\frac{2\nu_E}{f}\right)^{1/2}$: Ekman depth

b. The equations are

$$-fv = \nu_E \frac{\partial^2 u}{\partial z^2} \qquad \text{bound. conditions} \quad \nu_E \frac{\partial u}{\partial z} = \frac{\tau^x}{\rho_0}, \quad \nu_E \frac{\partial v}{\partial z} = \frac{\tau^y}{\rho_0} \quad z=0$$

$$fu = \nu_E \frac{\partial^2 v}{\partial z^2}$$

$$u, v \to 0 \quad \text{for} \quad z \to -\infty$$

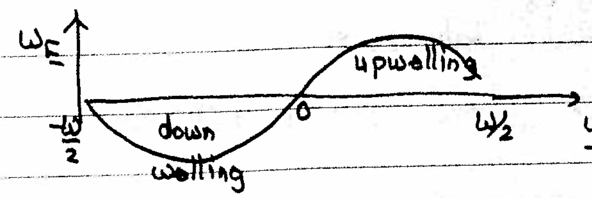
Note : no pressure gradient force here (no geostrophic flow)

Integrate and use $U = \int u\,dz$ and $V = \int v\,dz$

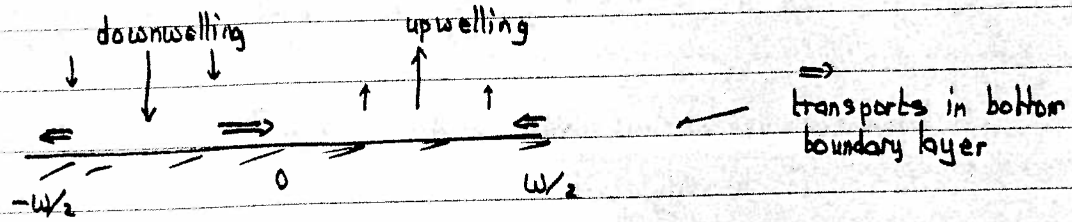
$$-fV = \nu_E \frac{\partial u}{\partial z}\Big|_{-\infty}^{0} = \frac{\tau^x}{\rho_0} \quad \text{so} \quad V = \frac{\tau_0 - \hat{\tau}}{\rho_0 f}$$

Likewise, $U = 0$

c. $$w_E = -\frac{1}{\rho_0 f}\frac{\partial \tau^x}{\partial y} = \frac{+\hat{\tau}}{\rho_0 f}\frac{2\pi}{W}\sin\left(\frac{2\pi y}{W}\right)$$



d.



So high pressure in souther region , low pressure in north

e. From eq. sheet : $$+\frac{d}{2}\bar{\zeta} = +\frac{1}{\rho_0 f}\left(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right)$$

Here $$-\frac{d}{2}\frac{\partial u_g}{\partial y} = -\frac{1}{\rho_0 f}\frac{\partial \tau^x}{\partial y}$$

So $$u_g = \frac{2\tau^x}{\rho_0 f d} + \text{constant} \quad \Rightarrow \quad \boxed{u_g = \frac{2}{\rho_0 f d}\hat{\tau}\cos\left(\frac{2\pi y}{W}\right)}$$

τ^x and apply $\langle u_g \rangle = 0$

$\Rightarrow$ westward flow near $y = \pm\frac{1}{2}W$, but eastward (opposing wind stress!) in central region

note : $\Delta v_g = 0$

f. $$\Gamma = \oint \vec{u}_g \cdot d\vec{\ell} \quad \to \quad \Gamma = u_g(0)L - u_g\left(\tfrac{W}{2}\right)L = \dots = \frac{4\hat{\tau}L}{\rho_0 f d}$$