MID-TERM EXAM GEOPHYSICAL FLUID DYNAMICS

9 November 2011, 9.00 - 11.00 hours

Two problems (all items have equal weight)

Remark: answers may be written in English or Dutch.

Problem 1

Consider the meridional momentum balance for a molecular viscous fluid, after application of the Boussinesq approximation:

$$\frac{dv}{dt} + f\,u = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right). \qquad (*)$$

a. Name all variables that appear in equation (*).
Also, specify the dimensional units of these variables.

b. Name and describe the physical meaning of all the terms that appear in equation (*).

c. Apply a Reynolds averaging procedure to equation (*).
Discuss the main steps of the procedure, present the equation for the resolved flow and indicate where the Reynolds stresses appear in the final equation.

d. Assume that the fluid is bounded from above by a free surface $z = \eta$.
Specify the boundary condition(s) at $z = \eta$ that correspond to equation (*) after application of the Reynolds procedure.

e. The meridional momentum balance for a Kelvin wave travelling along a coast located at $x = 0$ reads

$$\frac{\partial v}{\partial t} = -g\,\frac{\partial \eta}{\partial y}.$$

How, and under what conditions for key dimensionless numbers that characterise geophysical fluids, is this equation derived from equation (*)?

f. Sketch the structure of the free surface and the velocity components u, v for a Kelvin wave that travels along a coast that is located at $x = 0$ in the Northern Hemisphere.
Pay attention to the direction of propagation of the wave and its cross-shore structure.

For problem 2: P.T.O.

Problem 2

In the interior of a homogeneous fluid (constant depth) on the f-plane (Southern Hemisphere) the following pressure field exists:

$$p = \hat{p} \exp\left(-(x^2 + y^2)/L^2\right) + A\,y \qquad \text{for} \quad -2L \le y \le 2L\,.$$

Here, $\hat{p}$, A and L are constants.

a. Sketch pressure p as a function of x at $y = -L$, $y = 0$ and $y = L$ in one figure. Assume positive $\hat{p}$ and positive A

b. Compute expressions for the geostrophic flow components $\bar{u}$ and $\bar{v}$ in terms of the parameters $\hat{p}$, A and L.

c. Calculate the distribution of absolute vorticity in the interior of the fluid.

d. Is the absolute vorticity of fluid columns in the interior conserved while following the motion?
Explain your answer.

e. Compute the distribution of the Ekman pumping velocity $\bar{w}$ that is induced by the bottom boundary layer.
Also, sketch $\bar{w}$ at $y = 0$ as a function of x for $\hat{p} > 0$ and $A > 0$.

f. Sketch the horizontal velocity vector at $x = 0, y = 0$ for different z in the bottom boundary layer for $\hat{p} > 0$ and $A > 0$.
Explain your answer.

END

GFD 2011 Equation sheet

Continuity and momentum equations: molecular viscous fluid

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho\, u) + \frac{\partial}{\partial y}(\rho\, v) + \frac{\partial}{\partial z}(\rho\, w) = 0$$

$$\begin{aligned}
\rho\left(\frac{du}{dt} + f_* w - f\, v\right) &= -\frac{\partial p}{\partial x} && + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\
\rho\left(\frac{dv}{dt} + f\, u\right) &= -\frac{\partial p}{\partial y} && + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \\
\rho\left(\frac{dw}{dt} - f_* u\right) &= -\frac{\partial p}{\partial z} - \rho g && + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}$$

Relative circulation and relative vorticity

$$\Gamma = \oint_C \mathbf{u} \cdot d\mathbf{r} = \int_S (\nabla \times \mathbf{u}) \cdot \mathbf{n}\, dS$$

where S is the surface enclosed by contour C.

Shallow water equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - f\, v &= -g\frac{\partial \eta}{\partial x} \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f\, u &= -g\frac{\partial \eta}{\partial y} \\
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) &= 0
\end{aligned}$$

Ekman pump (Northern Hemisphere)

$$\overline{w} = \frac{d}{2}\overline{\zeta}, \qquad d = \left(\frac{2\nu_E}{f}\right)^{1/2} \qquad \text{and} \qquad w_{Ek} = \frac{1}{\rho_0 f}\left[\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right]$$