MID-TERM EXAM GEOPHYSICAL FLUID DYNAMICS (NS-353B)

7 November 2012, 9.00 - 11.00 hours

Two problems (all items have equal weight)

Remark 1: answers may be written in English or Dutch.

Remark 2: in all questions you may use $g = 10\ \mathrm{ms}^{-2}$ and $\Omega = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}$.

Remark 3: please write answers of the two problems on separate sheets.

Problem 1

Consider the continuity equation and momentum equations for a molecular viscous fluid on a rotating Earth, as are given on the supplementary equation sheet.

a. Indicate which term(s) include the effect of the centrifugal force that is caused by the spinning of the Earth around its axis. Motivate your answer.

b. Describe the Boussinesq approximation and use it to derive a reduced version of the meridional momentum balance.

c. Derive the meridional momentum balance that results from application of the Reynolds averaging procedure.
Also, identify and parameterise the Reynolds stresses.

d. In a homogeneous fluid, a particle (with the same density as that of the fluid) is released at vertical level $z = h$ on the equator. The initial velocity of the particle is $0.146\ \mathrm{ms}^{-1}$ in the vertical direction.
Neglecting friction and effects of dynamic pressure, compute the maximum vertical and horizontal displacement of the particle.
Hint: write down the equations of motion for u, v, w and solve them for the given initial conditions.

For problem 2: P.T.O.

Problem 2

An anticyclonic circular vortex in the ocean (depth $h = 2$ km) has a tangential velocity profile in its interior that is given by

$$v_\theta = \begin{cases} U\,\frac{r}{R} & \text{if } r \leq R, \\ 0 & \text{if } r > R. \end{cases}$$

In this expression, r, θ are polar coordinates. The corresponding Cartesian velocity components are $u = -\sin\theta\, v_\theta$ and $v = \cos\theta\, v_\theta$.

The radius $R = 100$ km and $U = 1\ \mathrm{ms}^{-1}$. The density is constant, and $|f| = 10^{-4}\ \mathrm{s}^{-1}$, $\nu_E = 10^{-2}\ \mathrm{m}^2\mathrm{s}^{-1}$.

a. On which hemisphere is the vortex located?
Explain your answer.

b. Give expressions for the Rossby number and the Ekman number and compute their values at location $x = 0, y = R/2$ of the vortex.

c. Use the geostrophic balance to derive an expression for the sea surface $\eta(r)$.
Assume the sea surface elevation to be zero at the boundary of the vortex.

d. Compute the absolute circulation of the vortex at its boundary.

e. Sketch the horizontal velocity vector at location $x = 0, y = R/2$ as a function of vertical coordinate z in the bottom Ekman layer.
Explain your answer.

END