FINAL EXAM GEOPHYSICAL FLUID DYNAMICS

3 November 2014, 9.00 - 11.00 (2 hours)

Two problems (all items have equal weight)

Remark 1: Answers may be written in English or Dutch. Please write clearly!

Remark 2: In all questions you may use $g = 10$ ms^{-2}, $r_a = 6400$ km and $\Omega = 7.3 \times 10^{-5}$ s^{-1}.

Remark 3: Please write your answers for each problem on a separate sheet of paper and put your name and student number on each sheet.

Problem 1

An underwater earthquake in a homogeneous, horizontally unbounded ocean of constant depth H generates an initial circular disturbance of the sea surface (height $\Delta\eta$, radius b). Thus

$$\eta = \begin{cases} \Delta\eta & \text{if } r < b\,, \\ 0 & \text{if } r > b\,, \end{cases}$$

in which $r^2 = x^2 + y^2$. The disturbance generates Poincaré waves that propagate away from the earthquake region. These waves are governed by the linear barotropic shallow water equations on the f-plane (see equation sheet) .

a. Discuss one similarity and two fundamental differences between Poincaré waves and Kelvin waves.

b. Assuming wave-like solutions

$$\begin{pmatrix} \eta \\ u \\ v \end{pmatrix} = \Re\left\{ \begin{pmatrix} A \\ U \\ V \end{pmatrix} e^{i(k_x x + k_y y - \omega t)} \right\},$$

derive expressions that relate each of the complex amplitudes U and V of Poincaré waves uniquely to A and the model parameters ω, k_x, k_y, g and f.

c. The dispersion relation of Poincaré waves is given by

$$\omega^2 = f^2 + gH(k_x^2 + k_y^2)\,.$$

Find an expression for the energy propagation speed (which is a scalar) for Poincaré waves with $\omega = 2f$ in terms of the two parameters g and H.

For rest of Problem 1 and Problem 2: P.T.O.

d. The Poincaré waves of items a-c provide the adjustment of the system to a steady end state. The latter has a free surface $\eta = \eta_e$ governed by

$$\nabla^2 \eta_e = \frac{f^2}{gH}(\eta_e - \Delta\eta) \qquad \text{for } r < d\,,$$

$$\eta_e = 0 \qquad \text{for } r > d\,,$$

where d is the position of the front. Derive these equations and write down the physical principles that you apply. For the second equation, use the appropriate boundary conditions.

e. Assuming, for simplicity, that $\nabla^2\eta_e = d^2\eta_e/dr^2$, solve the equations given in the previous item and derive an expression that determines the displacement of the front d from the original position of the disturbance b.

Problem 2

Consider flow that is governed by

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0\,,$$

$$q = \nabla^2\psi + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \psi}{\partial z}\right) + \beta_0\, y\,.$$

a. Name at least four conditions that are used to derive this equation from the full equations of motion that govern geophysical flows.

b. Show that the equation above admits a solution that represents a steady, zonal flow with an arbitrary meridional and vertical structure.

c. Assume a steady zonal flow in the atmosphere, which is governed by the equation given above, and which has a constant vertical shear α, i.e. $\frac{\partial u}{\partial z} = \alpha$, and no horizontal shear. Compute the density field that corresponds to this flow.

d. Under certain conditions, the steady flow of item c can be unstable with respect to small perturbations. Name this instability mechanism and discuss qualitatively whether this instability is enhanced, suppressed or not affected at all by

- an increase of parameter N;
- an increase of parameter β_0;
- an increase of parameter α.

Limit your answer to at most 0.5 page A4.

e. In the special case of zero steady background flow, the equations still allow for wave-like solutions. These waves have a dispersion relation as

$$\omega = \frac{-\beta_0 R^2 k_x}{1 + k^2 R^2}\,.$$

What are the names of these waves? Do these waves travel eastward or westward? And for the shorter waves ($|k_x| > R^{-1}$), in which direction does the energy of the waves propagate?

END