per 1+2

NS - 353b
prioriteit: 1

***Faculteit Natuur- en Sterrenkunde***
***BOZ/Julius Instituut***

*Tentamenvoorblad*
(gaarne zo volledig mogelijk invullen)

vak: NS-353B (Geofys. stromingsleer)

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d.d.: 31 januari 2007

van 14.00 uur tot 17.00 uur

in gebouw*: BBL zaal*: 107A

bijzonderheden:

open boektentamen: neen
formuleblad: neen
rekenmachine: neen

*) wordt door BOZ ingevuld

Geophysical Fluid Dynamics N/S-353B

Wednesday, January 31st 2007, 14:00-17:00

**All items have equal weight.**

# Problem 1

We study the propagation characteristics of planar Rossby waves in a reduced gravity quasi-geostrophic model. The potential vorticity equation reads:

$$\frac{dq}{dt_0} = 0 \tag{1}$$

with

$$q = \Delta_0\psi + f_0 + \beta y - \frac{1}{R_d^2}\psi \tag{2}$$

a) Explain the meaning of the terms in the expression for the potential vorticity, and the meaning of the first equation.

b) Give an expression for the Rossby deformation radius.

c) Linearize the equations around the rest state.

d) Find the dispersion relation for plane waves.

e) Derive expressions for phase and group velocity in the zonal direction.

f) Explain why the wave phase velocity has always a westward component, both for short and for long waves.

g) Sketch the angular frequency as function of the zonal wavenumber when the meridional wavenumber is zero. and discuss the meaning of the Rossby radius of deformation for the propagation of the wave.

h) Determine the minimum period of plane zonal Rossby waves in months at 35° north for the atmosphere, with $g' = 0.5\ ms^{-2}$, $H = 10\ km$, and for the ocean, with $g' = 0.01\ ms^{-2}$, $H = 900\ m$. Use $f = 1.0e^{-4}s^{-1}$, and $\beta = 2.0e^{-11}m^{-1}s^{-1}$.

# Problem 2

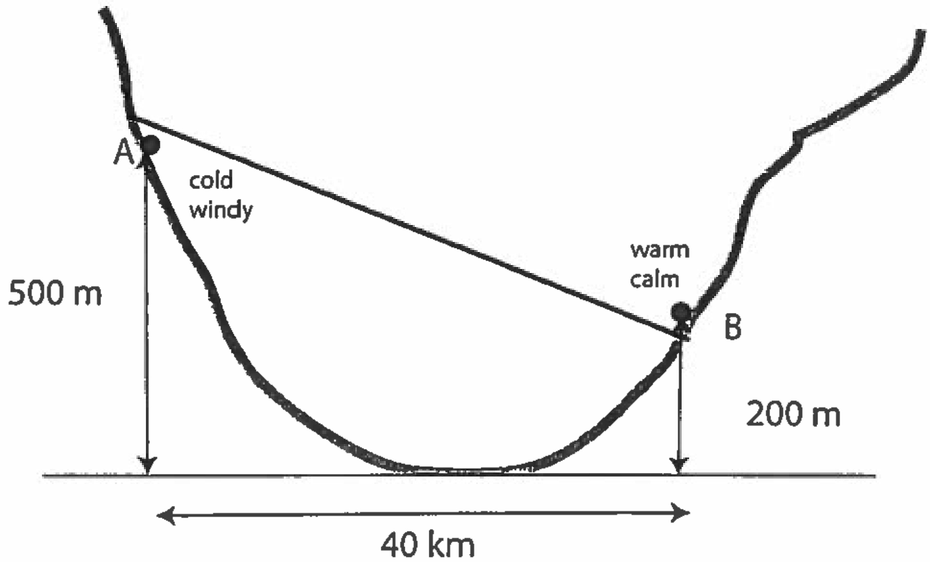


Figure 1: Situation sketch

In a meridionally oriented valley of the French Alps ($\approx 45^o\ N$) a village (A, see figure) is situated on a flank 500 $m$ above the valley floor. The weather is windy, with a temperature of $6^oC$. Another village (B) is situated on the other side of the valley 200 $m$ above the valley floor. The horizontal distance between the two villages is 40 $km$. Interestingly, the weather in village B is calm, with a temperature of $18^oC$! We assume that this situation is due to a cold wind blowing along one side of the valley.

a) Derive the thermal wind relation

$$p\frac{\partial v}{\partial p} = -\frac{R}{f}\frac{\partial T}{\partial x} \tag{3}$$

from geostrophy and hydrostatic balance in pressure coordinates:

$$fv = +\phi_x \qquad \phi_p = -\frac{RT}{p} \tag{4}$$

in which $\phi = gz$ is the geopotential, and $R = 287\ m^2 s^{-2} K^{-1}$.

b) What is the direction of the cold wind when the wind high up in the valley is zero?

Assuming a two-layer configuration in which the velocity and the temperature change discontinuously over the sloping interface from A to B (see figure), the thermal wind relation can be discretized as

$$p\frac{\Delta v}{\Delta p} = -\frac{R}{f}\frac{\Delta T}{\Delta x} \tag{5}$$

c) Use the hydrostatic balance to derive an equation for the wind in the lower layer in terms of the slope of the interface $\Delta z/\Delta x$ and estimate its strength.

# Problem 3

We study the stability of a two-layer flow on the $\beta$-plane. We start from the quasi-geostrophic potential vorticity equations in two-layers, given by:

$$\frac{dq_i}{dt_0} = 0 \tag{6}$$

for each layer $i$, with the potential vorticity given by:

$$q_1 = \Delta_0\psi_1 + \beta y - F(\psi_1 - \psi_2) \tag{7}$$

and

$$q_2 = \Delta_0\psi_2 + \beta y + F(\psi_1 - \psi_2) \tag{8}$$

in which the layers have equal undisturbed depth $H$, and

$$F = \frac{f_0^2}{g'H} \tag{9}$$

The basic flows inlayer 1 and 2 are zonal, and given by $U_1$ and $U_2$, respectively. $U_1$ and $U_2$ are constant in space and time, at least during the initial phases of instability that we study here.

a) Give 4 necessary conditions for instability of a baroclinic flow.

b) Use one of these to determine a necessary condition for the size of the vertical shear $|U_1 - U_2|$ for which the present basic flow is unstable.

c) Linearize the equations around the basic zonal flow with zonal velocities given by $U_1$ and $U_2$ in layer 1 and 2, respectively.

d) Look for wave-like solutions of the form $\phi_i = A_i \cos(lx + my - \omega t)$ and use them in the linearized equations. Explain how one *could* obtain an equation for the angular frequency $\omega$ (the dispersion relation) in which the amplitude of the wave does not appear.

e) Find a necessary condition for $(U_1 - U_2)^2$ for which the flow is unstable. Use that, after some algebra, the velocity $c = \omega/l$ is found from the dispersion relation as:

$$c = \frac{U_1 + U_2}{2} - \beta \frac{K^2 + F}{K^2(K^2 + 2F)} \pm \frac{\sqrt{4\beta^2 F^2 - K^4(U_1 - U_2)^2(4F^2 - K^4)}}{2K^2(K^2 + 2F)} \tag{10}$$

in which $K^2 = l^2 + m^2$. What is the condition on the wavenumber for unstable flow?

f) Why are these conditions also sufficient?

g) The lower bound for the vertical shear $|U_1 - U_2|$ that leads to instability is called the critical vertical shear. Find the wavenumber for which the critical vertical shear $|U_1 - U_2|$ is minimal, and determine this wavenumber and the corresponding wavelength when $f_0 = 1e^{-4}\ s^{-1}$, $g' = 0.025\ ms^{-2}$, and $H = 1000\ m$.

h) Determine this minimal critical vertical shear, using $\beta = 2e^{-11} m^{-1} s^{-1}$. How do the necessary and sufficient conditions for instability compare to the necessary condition in b)?

i) Explain the instability mechanism, ignoring potential vorticity changes due to $\beta$.