EXAM BLOCK 2 GEOPHYSICAL FLUID DYNAMICS

6 November 2018, 9:00 - 11:00 hours

Two problems (all items have equal weight)

Please write your answers for each problem on a separate sheet of paper and put your name and student number on each sheet.

Answers may be written in English or Dutch. Please write clearly!

In all questions you may use $g = 10$ ms^{-2}, $r_a = 6400$ km and $\Omega = 7.3 \times 10^{-5}$ s^{-1}.

Problem 1

Consider flow that is governed by the equations

$$\frac{\partial w}{\partial z} = \frac{1}{\rho_0 f_0^2}\left[\frac{\partial}{\partial t}\nabla^2 p' + \frac{1}{\rho_0 f_0} J(p', \nabla^2 p') + \beta_0 \frac{\partial p'}{\partial x}\right],$$

$$\frac{\partial \rho'}{\partial t} + \frac{1}{\rho_0 f_0} J(p', \rho') - \frac{\rho_0 N^2}{g} w = 0, \qquad N^2 = -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz}, \qquad \rho' = \frac{-1}{g}\frac{\partial p'}{\partial z}$$

a. What physical law(s) result in the equation that describes the time evolution of ρ'? Also, name the variables that appear in this equation.

b. Derive for this system expressions that relate horizontal density gradients to vertical shear of horizontal currents.
How are these relations called?

From now on, assume a system in which $\rho' = 0$. This results in the equation

$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi, \nabla^2\psi) + \beta_0 \frac{\partial \psi}{\partial x} = 0 \qquad (*)$$

where $\psi = p'/(\rho_0 f_0)$ is the stream function that only depends on x, y and t.

c. Show that equation (*) above admits a wave-like solution

$$\psi = \hat{\psi}\cos(k_x x + k_y y - \omega t),$$

with $\hat{\psi}$ a constant amplitude, and find the relation between ω, k_x and k_y.

d. For what values of k_x and k_y will the zonal energy propagation speed of a small-amplitude wave solution of item c be westward?

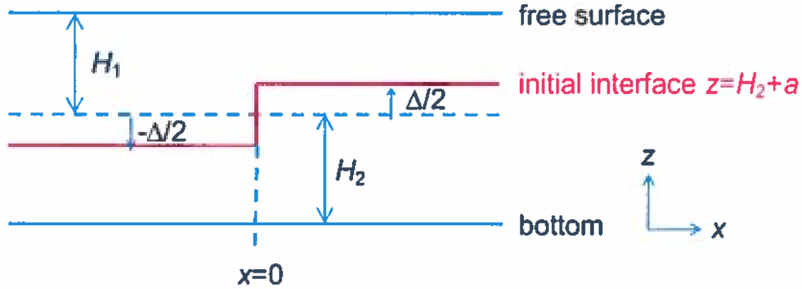
Note: if you do not have the answer to item c, use $\omega = \alpha k_x/(k_x^4 + k_y^4)$, where α is a positive constant.

e. Suppose one would like to verify the results of items c and d in a laboratory setting. Describe how such an experiment could be designed.

For problem 2: P.T.O.

Problem 2

Consider a two-layer fluid system on the f-plane. At time $t = 0$ there is no motion, whilst the interface between the layers has a distribution as is sketched in the figure below. Conditions in the y-directions are assumed to be uniform. The bottom is flat and horizontal.



It is assumed that at all times, the displacement a of the interface is much smaller than the undisturbed layer thicknesses H_1 and H_2, so linear equations can be used. Here, the focus is on the baroclinic response, which is described by the equations

$$\frac{\partial u_B}{\partial t} - f v_B = g' \frac{\partial a}{\partial x}, \qquad \frac{\partial v_B}{\partial t} + f u_B = 0,$$

$$-\frac{\partial a}{\partial t} + \bar{h} \frac{\partial u_B}{\partial x} = 0,$$

where $\bar{h}$ is defined on the equation sheet.

a. Why is it convenient to find solutions of the baroclinic mode (and barotropic mode) instead of deriving solutions of the original linear equations of the two-layer model, as are specified on the equation sheet?

b. Show that

$$\frac{\partial v_B}{\partial x} + \frac{f}{\bar{h}} a = F(x)$$

where $F(x) = \hat{F}$ for $x > 0$ and $F(x) = -\hat{F}$ for $x < 0$, with $\hat{F}$ a constant.
What is the physical interpretation of this result?

c. Show that in the steady end state the interface a is described by the differential equation

$$\frac{d^2 a}{dx^2} - \mu^2 a = \frac{-f}{g'} F(x).$$

Specify the constant μ in terms of the model parameters and give its physical meaning.

d. Find the solution $a(x)$ for the equation of item c and sketch $a(x)$.
Hint: use that both a and $\partial a/\partial x$ must be continuous for all x.

e. Is the the sum of kinetic and potential energy contents of the solution of item d smaller than, equal to, or larger than the total energy contents of the initial state?
Explain your answer.

END