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In 2005/2006, the course NS-353b was given by dr. P.J. van Leeuwen.

Geophysical Fluid Dynamics (NS-353b)
22 March 2006

Each item has equal weight.

Question 1

The primitive equations for the atmosphere are given by

$$\frac{\mathrm{d}u}{\mathrm{d}t} - fv = -\phi_x + \frac{1}{\rho}F^{(x)} \tag{1}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} + fu = -\phi_y + \frac{1}{\rho}F^{(y)} \tag{2}$$

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p} \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \tag{4}$$

$$\frac{\mathrm{d}\Theta}{\mathrm{d}t} = \frac{\Theta}{C_p}\frac{\mathrm{d}S}{\mathrm{d}t} \tag{5}$$

in the *isobaric* coordinate system in which the geopotential is given by $\phi = gz$, Θ is the potential temperature, the $F^{(i)}$ denote the friction, and the other variables are conventional.

a) Give the geostrophic relations and show which approximations are made in the horizontal momentum equations to obtain them.

b) Derive the thermal-wind relations by differentiating the geostrophic-balance relations w.r.t. the pressure. Use the hydrostatic equation to eliminate the geopotential.

c) The wind at the surface is from the west. At cloud level it is from the south. Do you expect the temperature to rise or fall?

d) The geostrophic wind will decrease due to friction at the bottom. Sketch the friction-induced secondary circulation of a cyclone in the vertical plane.

e) Why does the geostrophic velocity in the interior of the cyclone decrease?

f) What will happen to the temperature field in the interior?

Question 2

We consider waves in a barotropic fluid on an f plane with a sloping bottom. The layer thickness is given by $h = H - b$ with H constant, and $b = ax$, in which a is a positive constant. The quasi-geostrophic potential velocity equation reads:

$$\frac{\mathrm{d}q}{\mathrm{d}t} = 0, \tag{6}$$

in which

$$q = \Delta\psi + f_0\frac{b}{H} \tag{7}$$

with ψ the stream function.

a) Explain the physical meaning of the terms in the expression of the potential vorticity.

b) Linearize the potential vorticity equation around a state of rest.

c) Determine the dispersion relation for plane waves given by

$$\psi = Ae^{i(ly-\omega t)} \tag{8}$$

in which A is a constant, and find the phase and group velocity of these waves.

d) Explain why the phase velocity is positive on the northern hemisphere.

e) Determine the dispersion relation for plane waves when a background flow with constant meridional veocity V is present. How does this background flow influence phase and group velocity?

When the meridional background flow varies in strength in the zonal direction it can become unstable. A necessary condition for barotropic instability reads:

$$f_0 \frac{b_x}{H} + V_{xx} = 0 \tag{9}$$

f) Discuss the relation between this condition and the condition for barotropic instability on a β plane.

g) Explain the instability mechanism.

Question 3

We consider a barotropic fluid flowing along a continent on an f plane. The continent is located at $x = 0$. The velocity profile of the current is given by

$$v_1(x) = \begin{cases} V_1 \frac{x+L_1}{L_1} & \text{for } -L1 \le x \le 0 \\ 0 & \text{for } x \le -L1 \end{cases} \tag{10}$$

in which V_1 and L_1 are constants. The layer thickness of the fluid is given by $h_1(x) = -a_1 x$, with a_1 a constant. We want to predict the velocity of this current when the bottom changes slowly along the current path to a layer depth of $h_2(x) = -a_2 x$, with a_2 a constant.

a) Derive an expression for the relative vorticity structure of the current at h_2 in terms of that at h_1.

b) Determine the velocity structure of the current at that location. Use $v_2(x = -L_2) = 0$, in which L_2 a constant that still has to be determined.

c) Find an expression for L_2 from mass conservation.

d) Solve for L_2 assuming $f_0 = 10^{-4}\,\mathrm{s}^{-1}$, $V_1 = 2.5\,\mathrm{m\ s}^{-1}$, $L_1 = 50\,\mathrm{km}$ and $a_2 = 2a_1$.

e) What happens when $a_2 < 2/3a_1$, keeping the inflow at h_1 unchanged?