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In 2005/2006, the course NS-353B was given by P.J. van Leeuwen.

Geophysical Fluid Dynamics (NS-353B)
February 1st 2006

Each item has equal weight.

Question 1

We study waves in the ocean along the quator. To this end we use the linearized reduced-gravity model around the equator, given by:

$$
\begin{aligned}
u_t - \beta y v &= -g' \eta_x & (1)\\
v_t + \beta y u &= -g' \eta_y & (2)\\
\eta_t + H(u_x + v_y) &= 0 & (3)
\end{aligned}
$$

in which u and v are the zonal and meridional velocities, η is the interface elevation measured positive downward, and g' the reduced-gravity acceleration.

a) Explain which approximations have been made to obtain this set of equations from the primitive equations for the ocean.

We try to find wave-like solutions that have no meridional velocity component.

b) Derive the folowing wave equation from the zonal momentum equation and the continuity equation:

$$\eta_{tt} - g' H \eta_{xx} = 0 \qquad (4)$$

c) Determine the dispersion relation and the phase and group velocities of plane waves described by this equation.

d) Determine the meridional structure of the solutions from the meridional momentum equation. Use that the waves should vanish far from the equator. Show that the waves have to move eastwards.

e) Sketch the meridional momentum balance in the waves. Explain their propagation mechanism using geostrophy and the continuity equation.

Question 2

The quasi-geostrophic potential vorticity reads:

$$q = \Delta\psi + f_0 + \beta y + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\psi_z\right) \qquad (5)$$

a) Explain the physical meaning of the terms on the right-hand side of this equation.

b) In the quasi-geostrophic theory the horizontal divergence of the geostrophic velocity filed is zero. So, the continuity equation cannot be used to determine the vertical velocity. How is the vertical velocity calculated?

Show that

$$w = -\frac{d}{dt_0}\left(\frac{f_0}{N^2}\psi_z\right) \tag{6}$$

(Hint: Use geostrophy to relate ψ and p' and the hydrostatic balance to relate p' and ρ'.)

We now study the stability of a relatively simple flow in the quasi-geostrophic framework. Assume that the velocity profile is given by:

$$\overline{u} = -\psi_y = \frac{-U_0}{1 + y^2/2L^2} \tag{7}$$

in which U_0 is a, positive, constant in space and time.

c) Determine the minimal value for U_0 for which the flow can be unstable. Use $\beta = 2 * 10^{-11} m^{-1}s^{-1}$, and $L = 1000km$.

d) Will an eastward jet be more stable?

e) Give necessary conditions for instability for a stratified flow. Which one is used in item c)?

f) Show that even a mild zonal current that has a temperature at the ground that increases toward the equator can be unstable.

Question 3

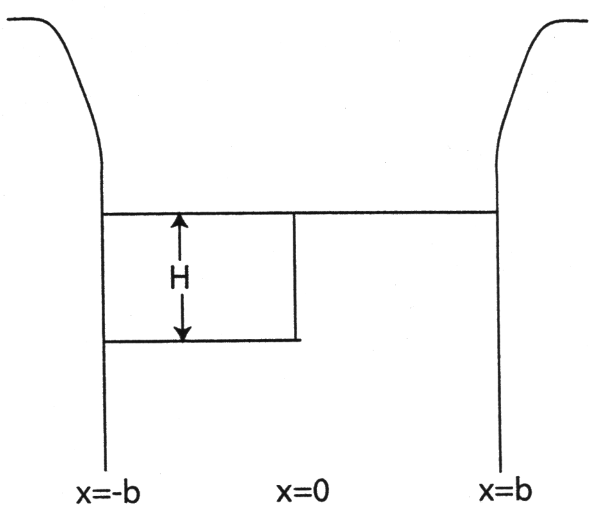

Figure 1: Zonal-depth section of the fjord before the adjustment process

A strong wind has set up a stratification in a narrow meridional fjord as depicted in the figure. The origin of the coordinate system is taken in the middle of the fjord, so that the two side walls are positioned at $x = -b$ and $x = b$. We want to predict the steady circulation in the fjord after the adjustment process that sets in. To this end, we approximate the system with a reduced gravity model on an f-plane. Because the fjord is very long we neglect all meridional dependence of the flow. This leads to:

$$\begin{aligned} u_t + uu_x - fv &= -g'\eta_x & (8)\\ v_t + uv_x + fu &= 0 & (9)\\ \eta_t + (hu)_x &= 0 & (10) \end{aligned}$$

a) Argue why the zonal velocity in the final state is zero.

b) Use the zonal momentum balance and conservation of potential vorticity to derive for the interface elevation in the final steady state:

$$\eta - R_d^2 \eta_{xx} = 0 \tag{11}$$

in which R_d is the Rossby radius of deformation.

c) Solve this equation when the light water reaches the right wall. Use volume conservation and $v(x = -b) = 0$.

d) What happens when $b \ll R_d$? Explain your answer.

We now study the case in which the light water does not reach the right wall. Assume that the upper-layer thickness becomes zero after adjustment at $x = a$, with $a < b$.

e) Show that from the boundary condition $v(x = -b) = 0$ and the vanishing upper-layer thickness at $x = a$ that

$$\eta = Ae^{x/R_d} + Be^{-x/R_d} \tag{12}$$

with

$$A = \frac{-H}{e^{a/R_d} + e^{-(a+2b)/R_d}} \tag{13}$$

and

$$B = Ae^{-2b/R_d} \tag{14}$$

in which H is the initial water depth.

f) Use the continuity equation to solve for a.

g) Study the solution when $b \gg R_d$.