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In 2005/2006, the course NS-HP428m was given by dr. J.B.M. Uffink.

Foundation of Quantum Mechanics (NS-HP428m)
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Question 1

a) What is meant by a density operator (also known as 'statistical operator' or 'state operator')?

b) Show that the class of density operators on a given Hilbert space forms a convex set. What are its extreme elements?

c) Show that the decomposition of a density operator in terms of its extreme elements is in general not unique. Discuss the implications of this fact for the interpretation of a density operator.

Question 2

Let $\mathcal{H}_1$ be the Hilbert space of an object system and Q an observable of this system. Let $\mathcal{H}_2$ be the Hilbert Space of a measuring apparatus, fit to measure observable Q, by means of a pointer observable R. We assume, for simplicity, that both Q and R are non-degenerate operators with discrete spectrum, and that $\dim \mathcal{H}_1 = \dim \mathcal{H}_2$ is finite.

a) What is the evolution of the composite system of object and measuring apparatus in an ideal measurement interaction according Von Neumann?

b) What is meant by the measurement problem in the wide and the strict sense?

c) Give a concise description of the "many worlds" interpretation of Everett, Wheeler & DeWitt, and discuss how the measurement problem is treated in this theory. Mention some strong and weak points of the view.

Question 3

Preparator A prepares a beam of electrons in the following manner. He has two devices. The first device produces electrons with spin up in the z-direction (state $|u\rangle$); the other device produces electrons with spin down (state $|v\rangle$). He tosses a fair coin to decide which device is to be used, and delivers the electron through a slit. This procedure is repeated arbitrarily often.
Preparator B has a source that produces pairs of electrons in a singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|u\rangle|v\rangle - |v\rangle|u\rangle). \tag{1}$$

He shields off one member of the pair and sends the other one through a slit. Again, the procedure is repeated arbitrarily often.

a) Give, for both preparation methods, the resulting (spin) state description in terms of a density operator.

b) Suppose an experimenter receives a beam of electrons, prepared either by A or by B. Can he distinguish the difference by performing experiments on this beam? Explain your answer.

c) Preparator C produces electrons in the state

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|u\rangle + |v\rangle) \tag{2}$$

Can the result of this preparation procedure be distinguished empirically from that of A? If so, give an example of an observable by means of which this distinction can be made.

Question 4

Give a succinct characterization of Bohr's interpretation of quantum mechanics. Pay particular attention to the concepts of "phenomenon" and "complementarity" in your answer.

Question 5

a) Describe the set-up of an EPR-experiment for spin-$\frac{1}{2}$ particles.

In a stochastic hidden-variables theory for this experiment, let outcomes be represented by $a, b \in \{-1, 1\}$, and the parameter settings by the variables α, β. It is assumed that there exists some hidden variable λ, such that one can specify, for each choice of the parameter settings (α, β), the conditional probability $p_{\alpha,\beta}(a, b|\lambda)$ of obtaining the pair of outcomes (a, b), when λ is given.
Since the variable λ might be 'hidden', and thus unknown, the correlations in the experiment are described by the following correlation function:

$$E(\alpha, \beta) := \sum_{a,b} ab \int p_{\alpha,\beta}(a, b|\lambda)\varrho_{\alpha,\beta}(\lambda)d\lambda, \tag{3}$$

where $\varrho_{\alpha,\beta}$ is some probability density over the hidden variable λ. Probability theory further provides the identity:

$$p_{\alpha,\beta}(a, b|\lambda) = p_{\alpha,\beta}(a|b, \lambda)p_{\alpha,\beta}(b|\lambda). \tag{4}$$

(Here $p_{\alpha,\beta}(b|\lambda) := \sum_b p_{\alpha,\beta}(a|b, \lambda)$, and $p_{\alpha,\beta}(a|b, \lambda)$ denotes the conditional probability of outcome a when outcome b and the hidden variable λ are both given.)
Now the following conditions are assumed (for all possible values of $a, b, \alpha, \beta, \lambda$:

$$p_{\alpha,\beta}(a|b, \lambda) = p_{\alpha,\beta}(a|\lambda) \tag{5}$$

$$p_{\alpha,\beta}(b|\lambda) = p_\beta(b|\lambda) \quad \text{and} \quad p_{\alpha,\beta}(a|\lambda) = p_\alpha(a|\lambda) \tag{6}$$

$$p_{\alpha,\beta}(\lambda) = \varrho(\lambda) \tag{7}$$

b) Discuss the physical motivations for the assumptions (5), (6), (7) in this experiment.

c) Show that the expected correlations (3) satisfy a Bell inequality:

$$|E(\alpha, \beta) + E(\alpha, \beta') + E(\alpha', \beta) - E(\alpha', \beta')| \leq 2 \tag{8}$$

d) Quantum mechanics can itself be construed as a "stochastic hidden variables" theory (where the quantum state playes the role of λ). Still, there are states for which

$$|E(\alpha, \beta) + E(\alpha, \beta') + E(\alpha', \beta) - E(\alpha', \beta')| = \sqrt{2} \tag{9}$$

Which of the assumptions (4), (5) and (6) are then violated?