NS-TP0401M

Dynamical Oceanography: 02-07-2010

Good Luck!

1. (30 points)

Consider a square ocean basin at mid-latitudes (Fig. 1). The basin has a constant depth D and the flow is forced by a wind stress field with a spatial pattern $\mathbf{T} = (\tau^x, \tau^y, 0)$.

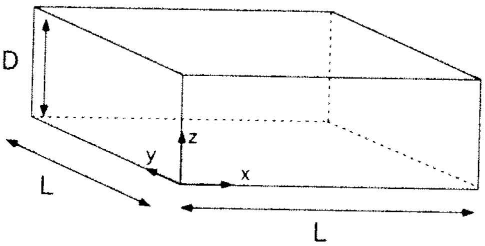

Figure 1: *Sketch of the model domain.*

Assume that the density ρ of the ocean water is constant, that the resulting flow is stationary and that effects of the deformation of the free surface can be neglected. In a quasi-geostrophic theory on the β-plane, the dimensionless potential vorticity equation for the geostrophic streamfunction ψ, on the flow domain $(x, y) \in [0, 1] \times [0, 1]$, is given by

$$\begin{aligned}(\frac{\delta_I}{L})^2(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y} &- \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi + \frac{\partial\psi}{\partial x} \\ &= \nabla\cdot(\mathbf{T}\wedge\mathbf{e}_3) - \frac{\delta_S}{L}\nabla^2\psi + (\frac{\delta_M}{L})^3\nabla^4\psi \qquad (1)\end{aligned}$$

with

$$(\frac{\delta_I}{L})^2 = \frac{1}{\beta} = (\frac{U}{\beta_0})^{1/2} \; ; \; (\frac{\delta_M}{L})^3 = \frac{1}{\beta Re} = (\frac{A_H}{\beta_0})^{1/3} \; ; \; \frac{\delta_S}{L} = \frac{r}{2\beta} = \frac{\delta_E f_0}{2D\beta_0} \qquad (2)$$

a. (5)
Provide the physical interpretation of each of the terms in equation (1).

Suppose from now on that $\delta_M = \delta_I = 0$.

b. (4)
Give the appropriate boundary conditions for the streamfunction ψ in this case.

The wind-stress field has the form

$$\tau^x = -\frac{1}{\pi} \cos \pi y \; ; \; \tau^y = 0 \qquad (3)$$

c. (6)
Determine the Sverdrup solution $\psi(x, y)$ that satisfies the boundary conditions at the eastern boundary.

d. (10)
Determine the Stommel western boundary layer solution $\psi_S(\lambda, y)$, where

$$\lambda = \frac{x}{l}.$$

and $l = \delta_S / L$.

e. (5)
Provide a description of the vorticity balances responsible for the fact that there is a western intensification of wind-driven ocean currents and not an eastern one.

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2. (30 points)

To investigate the adjustment of the midlatitude ocean circulation to a changing wind stress, a two-layer model is considered on a horizontally unbounded β−plane. In absence of flow, the layer thicknesses and densities are given by H_i and ρ_i, $i = 1, 2$, and the density difference $\Delta\rho = \rho_2 - \rho_1$ is much smaller than the reference density ρ_0; the reduced gravity $g' = g\Delta\rho/\rho_0$.

The interface between both layers, the thermocline, is described by $z_* = -h_* = D\ h$, where $D = H_1 + H_2$ is the total depth of the layer (Fig. 2). The bottom of the ocean is flat and both lateral and bottom friction can be neglected. In this case, the dimensionless equations for the two-layer model become

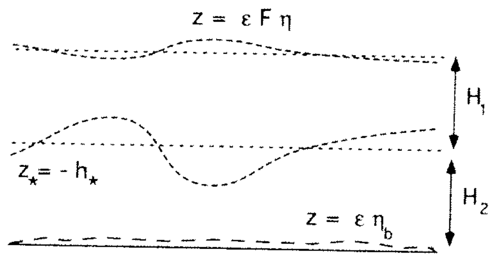


Figure 2: *Sketch of the two-layer model.*

$$\frac{D_1}{dt}\left[\nabla^2\psi_1 + \beta y + F_1(\psi_2 - \psi_1)\right] = \alpha\nabla\cdot(\mathbf{T}\wedge \mathbf{e}_3) + r_1\nabla^4\psi_1 \qquad (4a)$$

$$\frac{D_2}{dt}\left[\nabla^2\psi_2 + \beta y - F_2(\psi_2 - \psi_1)\right] = -r_2\nabla^2\psi_2 \qquad (4b)$$

Here, $\mathbf{T} = (\tau^x, \tau^y, 0)$ is the wind stress,

$$\frac{D_i}{dt} = \frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x} + v_i\frac{\partial}{\partial y} \qquad (5a)$$

$$u_i = -\frac{\partial\psi_i}{\partial y} \quad ; \quad v_i = \frac{\partial\psi_i}{\partial x} \qquad (5b)$$

$$h = \frac{H_1}{D} + \mu(\psi_1 - \psi_2) \qquad (5c)$$

In addition, r_1 and r_2 are dimensionless friction coefficients and

$$F_1 = \frac{f_0^2L^2}{g'H_1} \; ; \; F_2 = \frac{f_0^2L^2}{g'H_2} \; ; \; \mu = \frac{Uf_0L}{g'D} \qquad (6)$$

a. (4)
Give the physical interpretation of the parameters $F_i, i = 1, 2$.

b. (8)
Determine the dispersion relation for the pure free barotropic and pure free baroclinic Rossby wave in this model.

Assume that up to $t = 0$ the ocean is motionless

c. (6)
Describe the adjustment process to a steady wind-forced state when the wind forcing is turned on at $t = 0$.

Assume that after a while, a steady velocity field $(\bar{\psi}_1, \bar{\psi}_2)$ develops with

$$\bar{\psi}_1(y) = -\Phi(y) \ ; \ \bar{\psi}_2(y) = \gamma \bar{\psi}_1(y) \tag{7}$$

where Φ is a specific function and γ is a constant.

d. (6)
Under which conditions on Φ and γ can the flow in (7) become unstable through (i) pure barotropic instability and (ii) pure baroclinic instability?

e. (6)
Determine the thermocline shape h for the solution (7) with $\gamma = 1/2$ and with $\Phi(y) = y$ and describe the physical processes causing the meridional dependence of the thermocline.

3. (30 points)

A researcher performs a simulation with an idealized coupled ocean-atmosphere model of El Niño/Southern Oscillation (ENSO) covering the Tropical Pacific domain (from about 15°S to 15°N). Using scales

$$t_* = \frac{L}{c_o} t \; ; \; x_* = Lx \; ; \; y_* = \lambda_o y, \tag{8a}$$

$$h_* = Hh \; ; \; u_* = c_o u \; ; \; v_* = \frac{\lambda_o}{L} c_o v, \tag{8b}$$

$$c_o = \sqrt{g'H} \; ; \; \lambda_o = \sqrt{\frac{c_o}{\beta_0}}. \tag{8c}$$

the dimensionless ocean equations describing the free waves on a flat thermocline with equilibrium depth H in the reduced-gravity ocean model are:

$$\frac{\partial u}{\partial t} - yv + \frac{\partial h}{\partial x} = 0, \tag{9a}$$

$$(\frac{\lambda_0}{L})^2 \frac{\partial v}{\partial t} + yu + \frac{\partial h}{\partial y} = 0, \tag{9b}$$

$$\frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{9c}$$

a. (10)

There is only one type of free wave solution with $v = 0$. Determine the dispersion relation of this wave, its (meridional) spatial pattern and explain the physics of why this wave only propagates eastward.

The dimensional sea-surface temperature equation is given by

$$\frac{\partial T_*}{\partial t_*} + \mathbf{u}_* \cdot \nabla T_* = -\epsilon_T (T_* - T_0) - w_* \frac{T_* - T_{s*}(h_*)}{H} \tag{10}$$

where ϵ_T is a damping coefficient to an equilibrium temperature T_0 and $T_{s*}(h_*)$ is the sub-surface temperature.

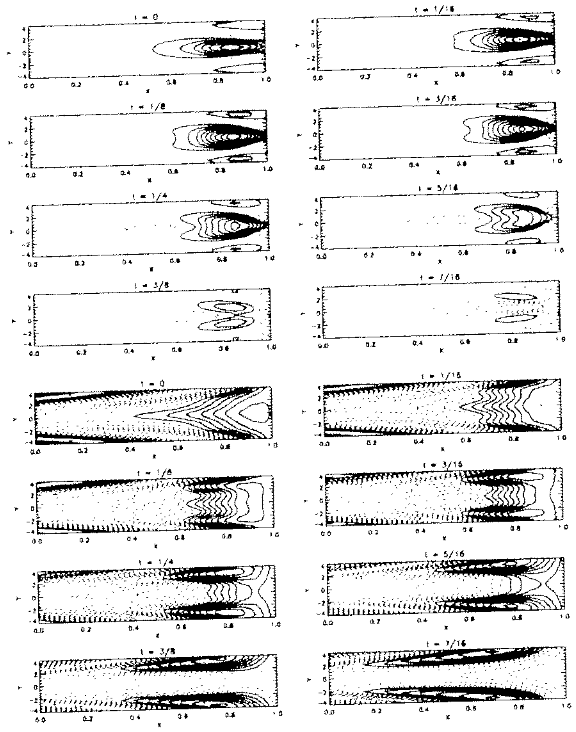
b. (5)

Argue why $T_{s*}(h_*)$ should be monotonic in h_* and sketch a typical shape of this function.

c. (5)

Describe the thermocline feedback with help of the sea surface temperature equation (10) above.

In Fig. 3, results of the simulation are plotted over nearly half an ENSO cycle. Panel (a) contains the pattern of the sea surface temperature anomalies (with respect to T_0) and panel (b) that of the thermocline depth anomalies (with respect to the depth H).



(a) (b)

Figure 3: *(a) Sea surface temperature anomaly and (b) Thermocline depth anomaly for half of an ENSO cycle. The time in the pictures is relative to a period of 3.7 years (in this model).*

d. (5)
What are characteristic amplitudes of the thermocline anomalies (in m) and temperature anomalies (in °C) during a typical El Niño (such as in 1997-1998) and how can one recognize the free equatorial waves in the thermocline anomalies shown in Fig. 3b?

e. (5)
With reference to both panels in Fig. 3, briefly describe the delayed oscillator mechanism of ENSO over half the period of the ENSO cycle shown.