Dynamical Oceanography: 30-06-2015

Much success!

1. (30 points)

Consider a square ocean basin at mid-latitudes (Fig. 1) as a model of the North Atlantic basin. The basin has a constant depth D and the flow is forced by a wind stress field with a spatial pattern $\mathbf{T} = (\tau^x, \tau^y, 0)$.

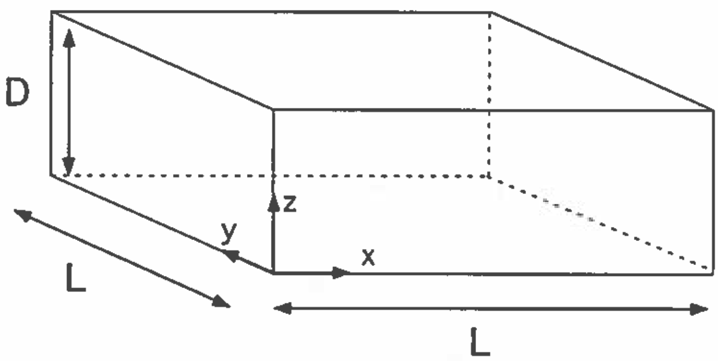


Figure 1: *Sketch of the model domain.*

Assume that the density ρ of the ocean water is constant and that the resulting flow is stationary. In a quasi-geostrophic (QG) theory on the β-plane, the steady **dimensional** vorticity equation for the geostrophic streamfunction ψ, on the flow domain $(x, y) \in [0, L] \times [0, L]$, is given by

$$\left(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)(\nabla^2\psi - \lambda_0\psi) + \beta_0\frac{\partial\psi}{\partial x} =$$
$$= \frac{1}{\rho_0 D}\nabla.(\mathbf{T}\wedge\mathbf{e}_3) - \epsilon_0\nabla^2\psi + A_H\nabla^4\psi \tag{1}$$

a. (6)

Give an expression for the QG potential vorticity (PV) in this model and a physical interpretation of each term in this expression?

b. (6)
Explain (in not more than 1/4 A4 text) how the wind stress forces the geostrophic flow (in this model) through processes in the Ekman layer.

Suppose from now on that the effects of bottom friction and inertia can be neglected over the whole flow domain (**there is lateral friction!**) and that $\lambda_0 = 0$. Assume furthermore that the wind-stress field has the form

$$\tau^x = -\frac{\tau_0}{\pi} \cos \frac{\pi y}{L} \; ; \; \tau^y = 0 \tag{2}$$

c. (8)
Determine the dimensional Sverdrup solution $\psi(x, y)$ that satisfies the boundary conditions at the eastern boundary.

d. (6)
Explain with help of vorticity balances of the flow, why the compensating return flow (with respect to the Sverdrup flow) can only occur on the western side of the basin and not on the eastern side.

e. (4)
Calculate the northward dimensional western boundary layer transport Φ through the section $y = L/4$.

2. (30 points)

Consider a flow in a zonal channel with constant depth D and width L, with $L >> D$, on a β-plane with $\theta_0 = 45^\circ$N. The horizontal velocity field is $\mathbf{v} = (u, v)$, the pressure p and the density ρ (see Fig. 2).

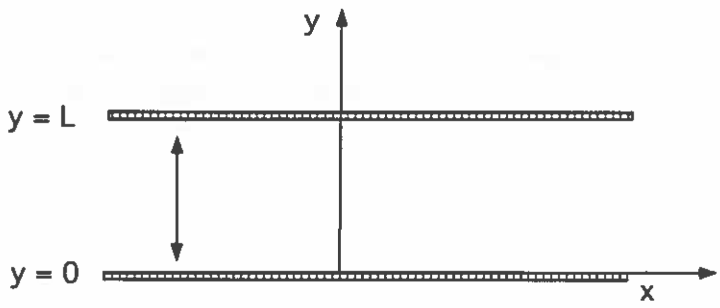


Figure 2: *Sketch of the flow domain in the zonal channel.*

Assume that the flow can be well-described by the **dimensionless** stratified quasi-geostrophic model as in the Appendix, with a constant Burger number S. Under a specified wind forcing, a steady flow is realized with a dimensionless velocity field (on the domain $y \in [-1, 1]$, $z \in [-1, 0]$) given by

$$\bar{u}(y,z) = U(y)(\eta_3 z + 1) \quad (3a)$$
$$\bar{v}(y,z) = 0 \quad (3b)$$

where $U(y) = \eta_1(1 + \eta_2 \cos \pi y)$ and the η_i $i = 1, 2, 3$ are constants.

a. (8)
Determine and sketch the dynamic density field $\bar{\rho}$ for the flow (3) in the case $\eta_1 = 1, \eta_2 = 1, \eta_3 = 1$.

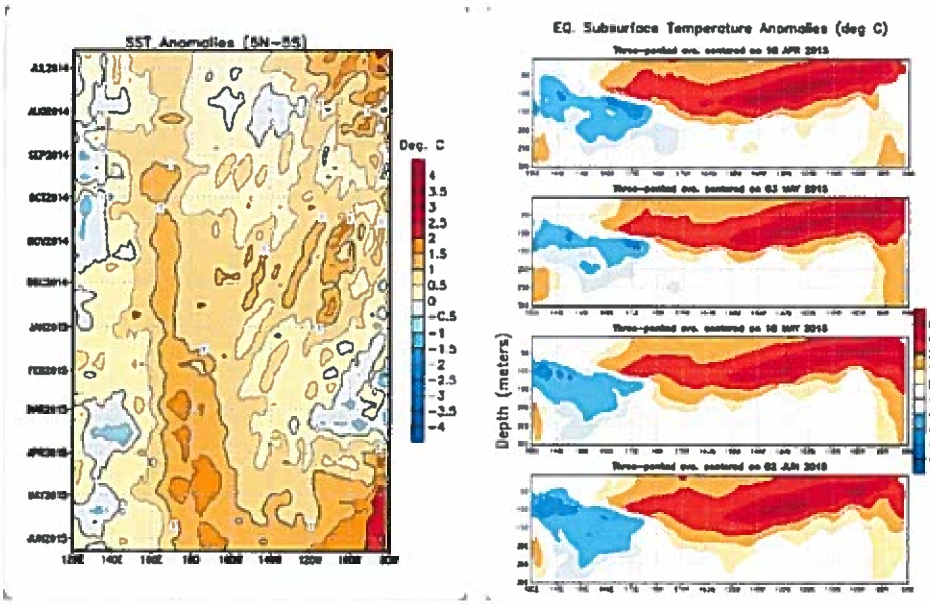
b. (8)
For $\eta_2 = 1, \eta_3 = 0$, determine conditions on η_1 for which the flow is barotropically stable. Briefly describe the mechanism of barotropic instability in the case $\eta_1 < 0$.

c. (8)
Determine the conditions on the constants η_i for which the flow is baroclinically stable. What is the main difference between the baroclinic and barotropic instability mechanisms in terms of energy transfer between the steady flow and the perturbations?

d. (6)
Describe the transient flow (the adjustment) which will occur when a stationary state for $\eta_1 = 1, \eta_2 = 1, \eta_3 = 0$ undergoes a change in the wind forcing at $t = 0$.

3. (30 points)

In the equatorial Pacific, the equatorial sea surface temperature (SST) anomalies are usually taken over the latitudes [2°S - 2°N]. In Fig. 3a, a time-longitude diagram of the equatorial SST anomalies is shown over the period June 2014- June 2015. In Fig. 3b, a longitude-vertical section of the upper ocean temperature anomalies is plotted for four periods between April 2015 and June 2015.



(a) (b)

Figure 3: *(a) Time-longitude diagram of equatorial SST anomalies (°C) over the period June 2014 - June 2015. (b) Longitude-depth section of equatorial temperature anomalies for four recent periods.*

a. (5)

Explain (briefly) why the sub-surface temperature anomalies tend towards the surface in the eastern part of the basin (as seen in Fig. 3b).

A researcher wants to understand the eastward propagation of the SST anomalies as seen in Fig. 3. The **dimensional** ocean equations describing small amplitude motions on a flat thermocline with equilibrium depth H in a reduced-gravity ocean model are

$$\frac{\partial u}{\partial t} - \beta_0 y v + g' \frac{\partial h}{\partial x} = 0, \tag{4a}$$
$$\frac{\partial v}{\partial t} + \beta_0 y u + g' \frac{\partial h}{\partial y} = 0, \tag{4b}$$
$$\frac{\partial h}{\partial t} + H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0, \tag{4c}$$

where (u, v) is the horizontal velocity anomaly and h the thermocline anomaly.

b. (10)
Determine the dimensional solution (u, v, h) for the wave which causes this eastward propagation, derive its dispersion relation and explain why this wave cannot propagate westward.

The dimensional sea surface temperature equation is given by

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = -\epsilon_T (T - T_0) - w \frac{T - T_s}{H} \tag{5}$$

where ϵ_T is a damping coefficient to a constant radiation equilibrium temperature T_0 and T_s is the sub-surface temperature.

c. (4)
Give a plausible functional form of the sub-surface temperature $T_s(h)$ and provide a physical motivation for this function.

d. (5)
Give a description of the zonal advection feedback with help of the sea surface temperature equation (5) above.

e. (6)
Briefly describe the mechanism of the El Niño/Southern Oscillation phenomenon assuming that the thermocline feedback is the dominant Bjerknes' feedback. Start the description with a positive SST anomaly in the eastern Pacific.

Appendix
The stratified quasi-geostrophic model on the β-plane

The $\mathcal{O}(1)$ equations are

$$
\begin{aligned}
v^0 &= \frac{\partial p^0}{\partial x} && (6a)\\
u^0 &= -\frac{\partial p^0}{\partial y} && (6b)\\
0 &= -\frac{\partial p^0}{\partial z} - \rho^0 && (6c)\\
0 &= \frac{\partial u^0}{\partial x} + \frac{\partial v^0}{\partial y} + \frac{\partial w^0}{\partial z} && (6d)
\end{aligned}
$$

The quasi-geostrophic vorticity equation (with $\psi = p^0$) is

$$
\left(\frac{\partial}{\partial t} + u^0\frac{\partial}{\partial x} + v^0\frac{\partial}{\partial y}\right)\left(\nabla^2\psi + \frac{\partial}{\partial z}\left(\frac{1}{S}\frac{\partial\psi}{\partial z}\right) + \beta y\right) = 0 \qquad (7)
$$

The boundary condition for ψ at $z = -1$ is

$$
w^1 = -\frac{1}{S}\left(\frac{\partial}{\partial t} + u^0\frac{\partial}{\partial x} + v^0\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial z} = 0 \qquad (8)
$$

and the boundary condition at the ocean-atmosphere interface $z = 0$ is

$$
w^1 = -\frac{1}{S}\left(\frac{\partial}{\partial t} + u^0\frac{\partial}{\partial x} + v^0\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial z} = \frac{\alpha r}{2}\nabla.(\mathbf{T} \wedge \mathbf{e}_3) \qquad (9)
$$

with

$$
S = \frac{N^2 D^2}{f_0^2 L^2}\ ;\ \beta = \frac{\beta_0 L^2}{U} \qquad (10)
$$