Dynamical Oceanography: Final (04-06-2009)

Good Luck!

1. (30 points)

Consider a square ocean basin at mid-latitudes (Fig. 1). The basin has a constant depth D and the flow is forced by a wind stress field with a spatial pattern $\mathbf{T} = (\tau^x, \tau^y, 0)$.

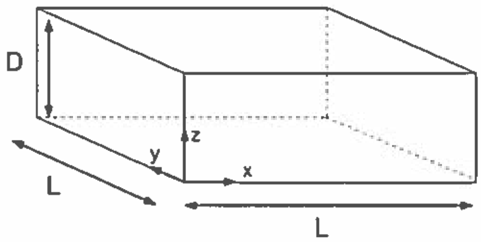


Figure 1: *Sketch of the model domain.*

Assume that the density ρ of the ocean water is constant, that the resulting flow is stationary and that effects of the deformation of the free surface can be neglected. In a quasi-geostrophic theory on the β-plane, the dimensionless potential vorticity equation for the geostrophic streamfunction ψ, on the flow domain $(x, y) \in [0, 1] \times [0, 1]$, is given by

$$\begin{aligned}(\frac{\delta_I}{L})^2(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y} &- \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x})\nabla^2\psi + \frac{\partial\psi}{\partial x} \\ &= \nabla.(\mathbf{T} \wedge \mathbf{e}_3) - \frac{\delta_S}{L}\nabla^2\psi + (\frac{\delta_M}{L})^3\nabla^4\psi \qquad (1)\end{aligned}$$

with

$$(\frac{\delta_I}{L})^2 = \frac{1}{\beta} = (\frac{U}{\beta_0})^{1/2} \; ; \; (\frac{\delta_M}{L})^3 = \frac{1}{\beta Re} = (\frac{A_H}{\beta_0})^{1/3} \; ; \; \frac{\delta_S}{L} = \frac{r}{2\beta} = \frac{\delta_E f_0}{2D\beta_0} \quad (2)$$

a. (5)
Provide the physical interpretation of the quantity δ_M.

Suppose from now on that the effects of lateral friction and inertia can be neglected over the whole flow domain. The wind-stress field has the form

$$\tau^x = -\frac{1}{k\pi}\cos k\pi y \; ; \; \tau^y = 0 \tag{3}$$

for certain integer $k > 0$.

b. (10)
For $k = 2$ in (3), determine the Sverdrup solution $\psi(x, y)$ that satisfies the kinematic boundary condition at the eastern boundary.

c. (10)
For $k = 2$ in (3), determine the Stommel western boundary layer solution $\psi_S(\lambda, y)$, where

$$\lambda = \frac{x}{\delta_S/L}.$$

d. (5)
Determine the values of k in (3) for which the total input of vorticity (by the wind stress) over the basin is zero.

2. (30 points)

To investigate the time-dependent behavior of the midlatitude ocean circulation, a two-layer model is considered on a horizontally unbounded β−plane. In absence of flow, the layer thicknesses and densities are given by H_i and ρ_i, $i = 1, 2$, and the density difference $\Delta\rho = \rho_2 - \rho_1$ is much smaller than the reference density ρ_0; the reduced gravity $g' = g\Delta\rho/\rho_0$.

The interface between both layers, the thermocline, is described by $z_* = -h_* = D\ h$, where $D = H_1 + H_2$ is the total depth of the layer (Fig. 2). The bottom of the ocean is flat and both lateral and bottom friction can

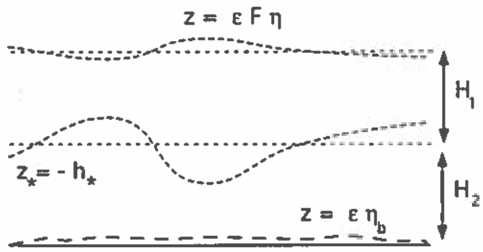


Figure 2: *Sketch of the two-layer model.*

be neglected. In this case, the dimensionless equations for the unforced, nondissipative two-layer model become

$$\frac{D_1}{dt}\left[\nabla^2\psi_1 + \beta y + F_1(\psi_2 - \psi_1)\right] = 0 \tag{4a}$$

$$\frac{D_2}{dt}\left[\nabla^2\psi_2 + \beta y - F_2(\psi_2 - \psi_1)\right] = 0 \tag{4b}$$

with

$$\frac{D_i}{dt} = \frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x} + v_i\frac{\partial}{\partial y} \tag{5a}$$

$$u_i = -\frac{\partial\psi_i}{\partial y} \quad ; \quad v_i = \frac{\partial\psi_i}{\partial x} \tag{5b}$$

$$h = \frac{H_1}{D} + \mu(\psi_1 - \psi_2) \tag{5c}$$

and

$$F_1 = \frac{f_0^2 L^2}{g' H_1} \ ; \ F_2 = \frac{f_0^2 L^2}{g' H_2} \ ; \ \mu = \frac{U f_0 L}{g' D} \tag{6}$$

a. (6)
Give the expression of the internal Rossby deformation radius L_{D1}, based on the first layer, and give a physical interpretation of this length scale.

The velocity field $(\bar{\psi}_1, \bar{\psi}_2)$ with

$$\bar{\psi}_1(y) = -U_1 y \ ; \ \bar{\psi}_2(y) = \gamma \bar{\psi}_1(y) \tag{7}$$

where U_1 and γ are constant, is a solution of the equations (4), *no need to check this.*

b. (8)
Determine the thermocline shape h for the solution (7) with $\gamma = 1/2$ and describe the physical processes causing the meridional dependence of the thermocline.

c. (8)
Answer and explain: can the flow in (7) become unstable through (i) barotropic instability and/or (ii) baroclinic instability?

Next, small amplitude motions are assumed on the solution (7) and wave solutions are considered.

d. (8)
Determine for $\gamma = 1$ the dispersion relation of the baroclinic mode.

3. (30 points)

A researcher performs a simulation with an idealized coupled ocean-atmosphere model of El Niño/Southern Oscillation (ENSO) covering the Tropical Pacific domain (from about 15°S to 15°N). Using scales

$$t_* = \frac{L}{c_o}t \ ; \ x_* = Lx \ ; \ y_* = \lambda_o y, \tag{8a}$$

$$h_* = Hh \ ; \ u_* = c_o u \ ; \ v_* = \frac{\lambda_o}{L}c_o v, \tag{8b}$$

$$c_o = \sqrt{g'H} \ ; \ \lambda_o = \sqrt{\frac{c_o}{\beta_0}}. \tag{8c}$$

the dimensionless ocean equations describing the free waves on a flat thermocline with equilibrium depth H in the reduced-gravity ocean model are:

$$\frac{\partial u}{\partial t} - yv + \frac{\partial h}{\partial x} = 0, \tag{9a}$$

$$\left(\frac{\lambda_0}{L}\right)^2\frac{\partial v}{\partial t} + yu + \frac{\partial h}{\partial y} = 0, \tag{9b}$$

$$\frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{9c}$$

a. (10)

There is only one type of free wave solution with $v = 0$. Determine the dispersion relation of these waves and determine their (meridional) spatial pattern.

The dimensional sea-surface temperature equation is given by

$$\frac{\partial T_*}{\partial t_*} + \mathbf{u}_* \cdot \nabla T_* = -\epsilon_T(T_* - T_0) - w_*\frac{T_* - T_{s*}(h_*)}{H} \tag{10}$$

where ϵ_T is a damping coefficient to an equilibrium temperature T_0 and $T_{s*}(h_*)$ is the sub-surface temperature.

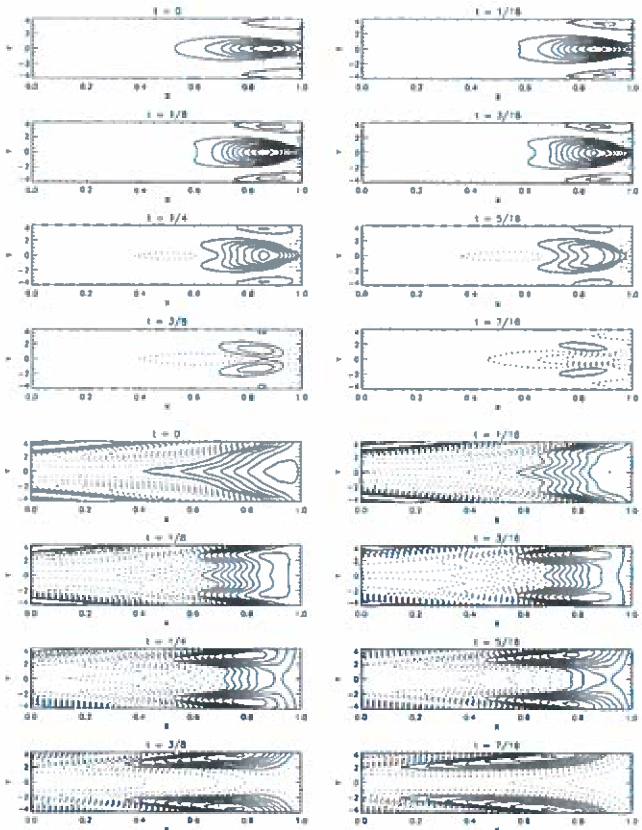
b. (5)

Sketch a reasonable shape of the function $T_{s*}(h_*)$ and motivate your answer.

c. (5)

Describe the thermocline feedback with help of the sea surface temperature equation (10) above.

In Fig. 3, results of the simulation are plotted over nearly half an ENSO cycle. Panel (a) contains the pattern of the sea surface temperature and panel (b) that of the thermocline depth anomaly (with respect to the depth H).



(a) (b)

Figure 3: *(a) Sea surface temperature anomaly and (b) Thermocline depth anomaly for half of an ENSO cycle. The time in the pictures is relative to a period of 3.7 years (in this model).*

d. (5)

Explain the physics of the eastern thermocline changes (as in Fig. 3b) between the times $t = 0$ and $t = 7/16$.

e. (5)

With reference to both panels in Fig. 3, briefly describe the ENSO mechanism over the half period of the ENSO cycle shown.