Dynamical Oceanography: 22-06-2017

Much success!

1. (30 points)

Consider a square ocean basin at mid-latitudes (Fig. 1) as a model of the North Atlantic basin. The basin has a constant depth D and the flow is forced by a wind stress field with a spatial pattern $\mathbf{T} = (\tau^x, \tau^y, 0)$.

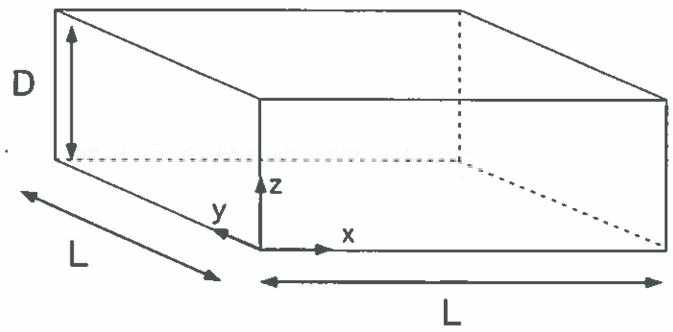


Figure 1: *Sketch of the model domain.*

Assume that the density ρ of the ocean water is constant and that the resulting flow is stationary. In a quasi-geostrophic (QG) theory on the β-plane, the steady **dimensional** vorticity equation for the geostrophic streamfunction ψ, on the flow domain $(x, y) \in [0, L] \times [0, L]$, is given by

$$\left(\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\right)(\nabla^2\psi - \lambda_0\psi) + \beta_0\frac{\partial \psi}{\partial x} = \frac{1}{\rho_0 D}\nabla.(\mathbf{T} \wedge \mathbf{e}_3) - \epsilon_0\nabla^2\psi + A_H\nabla^4\psi \quad (1)$$

a. (4)

Give an expression of the **dimensional** QG potential vorticity (PV) in this model. Under which conditions is PV conserved?

b. (6)

Consider the case that the effects of inertia and bottom friction can be neglected over the whole flow domain and that $\lambda_0 = 0$. In this case, the flow domain can be divided into two regions where different dominant vorticity balances hold. Sketch the different domains and provide the vorticity balance in each domain.

Suppose from now on that the effects of lateral friction and inertia can be neglected over the whole flow domain (**there is bottom friction!**) and that $\lambda_0 = 0$. Assume furthermore that the wind-stress field has the form

$$\tau^x = -\tau_0\left[\frac{(1-\sigma)}{\pi}\cos\frac{\pi y}{L} + \frac{\sigma}{2\pi}\cos\frac{2\pi y}{L}\right] \; ; \; \tau^y = 0 \quad (2)$$

where $\sigma \in [0, 1]$ is a real parameter.

c. (10)

Determine the **dimensional** Sverdrup solution $\psi(x, y)$ that satisfies the kinematic boundary condition at the eastern boundary.

d. (6)

Consider the case $\sigma = 0$. Explain with the help of PV budgets why the compensating return flow (with respect to the Sverdrup flow) can only occur on the western side of the basin and not on the eastern side.

e. (4)

For a range of σ there exists a latitude $y_0 \in (0, L)$ for which the meridional volume transport of the western boundary current is zero. Determine the dependence of this latitude y_0 on σ.

2. (30 points)

Consider a flow in a zonal channel with constant depth D and width L, with $L >> D$ on a β-plane with $\theta_0 = 45°$N. The horizontal velocity field is $\mathbf{v} = (u, v)$, the pressure p and the density ρ (see Fig. 2).

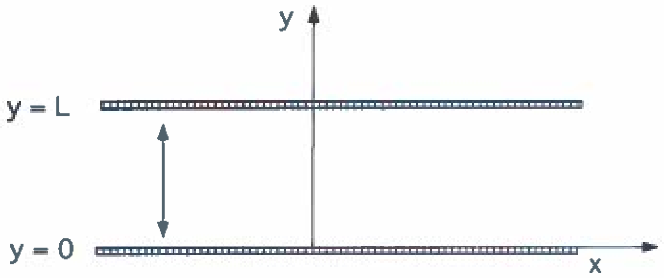


Figure 2: *Sketch of the flow domain in the zonal channel.*

Assume that the flow can be well-described by the **dimensionless** stratified quasi-geostrophic model as in the Appendix, with a constant Burger number S. Under a certain wind forcing, a steady flow is realized with dimensionless velocity field (on the domain $y \in [-1, 1]$, $z \in [-1, 0]$)

$$\bar{u}(y, z) = U(1 - \eta y^2)(z + 1) \quad (3a)$$
$$\bar{v}(y, z) = 0 \quad (3b)$$

where $U \geq 0$ is a constant.

a. (8)
Determine and sketch the three-dimensional steady density field $\bar{\rho}(x, y, z)$ for the flow (3) in the case $\eta = 0$.

b. (8)
Can the flow in (3) for the case $\eta = 1$ become unstable to (i) barotropic instability or (ii) baroclinic instability? Provide a short description of both instability mechanisms and use that to clarify your answer to (i) and (ii) above.

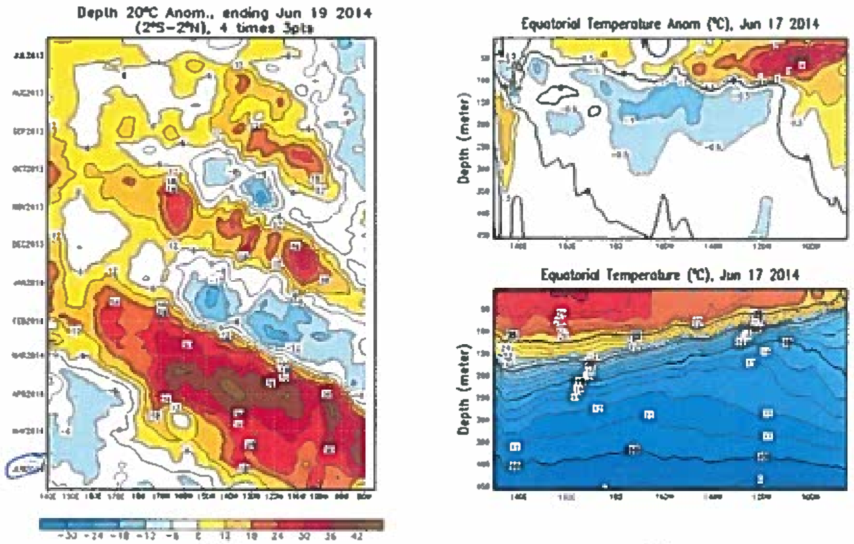
In the zonal channel, the meridional structure of a free Rossby wave is of the form $\cos[(n + 1/2)\pi y]$, for $n = 0, 1, 2, \ldots$.

c. (10)
Determine for $U = 0$, the zonal phase velocity of free barotropic Rossby waves in this model. Shortly describe the mechanism of propagation of the $n = 0$ barotropic Rossby wave.

d. (4)
Describe the adjustment process for the case $\eta = 0$ when the wind forcing is changed such that U suddenly becomes $U/2$.

3. (30 points)

In the equatorial Pacific, the depth of the 20°C isotherm is usually taken as the thermocline depth. In Fig. 3a, a time-longitude diagram of the equatorial (average over [2°S, 2°N]) thermocline anomalies is shown over the period June 2013- June 2014. In Fig. 3b, the equatorial temperature (lower panel) and the anomalies (upper panel) are plotted for June 17.



(a) (b)

Figure 3: *(a) Time-longitude diagram of equatorial thermocline anomalies (m) over the period June 2013 - June 2014. (b) Equatorial temperature (lower panel) and temperature anomalies (upper panel) for June 17.*

a. (5)
Give a short explanation for the shape of the thermocline in the lower panel of Fig. 3b.

A student wants to understand the eastward propagation of the thermocline anomalies as seen in Fig. 3a. The **dimensional** ocean equations describing small amplitude motions on a flat thermocline with equilibrium depth H in a reduced-gravity ocean model are

$$\frac{\partial u}{\partial t} - \beta_0 y v + g'\frac{\partial h}{\partial x} = 0, \quad (4a)$$

$$\frac{\partial v}{\partial t} + \beta_0 y u + g'\frac{\partial h}{\partial y} = 0, \quad (4b)$$

$$\frac{\partial h}{\partial t} + H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0, \quad (4c)$$

where (u, v) is the horizontal velocity anomaly and h the thermocline anomaly.

b. (10)
Determine the dimensional solution (u, v, h) for the wave which causes this eastward propagation, derive its dispersion relation and explain why this wave cannot propagate westward.

The dimensional sea-surface temperature equation is given by

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = -\epsilon_T(T - T_0) - w\frac{T - T_s}{H} \quad (5)$$

where ϵ_T is a damping coefficient to a constant radiation equilibrium temperature T_0 and T_s is the sub-surface temperature.

c. (4)
Explain where the negative subsurface temperature anomalies in equatorial Pacific (at about 150 m depth) in the upper panel of Fig. 3b originate from.

d. (5)
Give a description of the thermocline feedback with help of the sea surface temperature equation (5) above.

e. (6)
Explain the occurrence of the strong eastward propagating SST anomalies over the period February 2014 to May 2014.

Appendix
The stratified quasi-geostrophic model on the β-plane

The $\mathcal{O}(1)$ equations are

$$v^0 = \frac{\partial p^0}{\partial x} \quad (6a)$$

$$u^0 = -\frac{\partial p^0}{\partial y} \quad (6b)$$

$$0 = -\frac{\partial p^0}{\partial z} - \rho^0 \quad (6c)$$

$$0 = \frac{\partial u^0}{\partial x} + \frac{\partial v^0}{\partial y} + \frac{\partial w^0}{\partial z} \quad (6d)$$

The quasi-geostrophic vorticity equation (with $\psi = p^0$) is

$$\left(\frac{\partial}{\partial t} + u^0\frac{\partial}{\partial x} + v^0\frac{\partial}{\partial y}\right)\left(\nabla^2\psi + \frac{\partial}{\partial z}\left(\frac{1}{S}\frac{\partial\psi}{\partial z}\right) + \beta y\right) = 0 \quad (7)$$

The boundary condition for ψ at $z = -1$ is

$$w^1 = -\frac{1}{S}\left(\frac{\partial}{\partial t} + u^0\frac{\partial}{\partial x} + v^0\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial z} = 0 \quad (8)$$

and the boundary condition at the ocean-atmosphere interface $z = 0$ is

$$w^1 = -\frac{1}{S}\left(\frac{\partial}{\partial t} + u^0\frac{\partial}{\partial x} + v^0\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial z} = \frac{\alpha r}{2}\nabla.(\mathbf{T}\wedge\mathbf{e}_3) \quad (9)$$

with

$$S = \frac{N^2 D^2}{f_0^2 L^2}\ ;\ \beta = \frac{\beta_0 L^2}{U} \quad (10)$$