Exam: Dynamical Meteorology
Date: March, 19, 2010, 09:00-12:00.

In this exam all symbols have their normal definitions.

Problem 1 (2 points)

Invertibility principle

(a) Describe in words the implications of the invertibility principle for potential vorticity.

(b) What is the dynamical significance of the Rossby deformation height,

$$\Delta z = \frac{\sqrt{f(f+\zeta)}}{N}$$

(N is the Brunt-Väisälä frequency)?

Problem 2 (0.5 points)

Multiple choice

Which of the three possibilities is correct?

When a rotating homogeneous layer of fluid adjusts to geostrophic balance after being disturbed, potential energy is released. Most of this potential energy is carried away by the waves if

(a) the horizontal scale of the initial disturbance is large compared to the Rossby radius of deformation

(b) the horizontal scale of the initial disturbance is small compared to the Rossby radius of deformation

(c) the horizontal scale of the initial disturbance is of the same order of magnitude as the Rossby radius of deformation.

Problem3 (3 points)

Bariclinic waves

The dispersion relation for waves of the form $A\exp[ik(x-ct)]$ in the two-layer quasi-geostrophic model is

$$c = U_m - \frac{\beta\left(k^2+\lambda^2\right)}{k^2\left(k^2+2\lambda^2\right)} \pm \delta^{1/2}, \qquad (1)$$

with

$$\delta \equiv \frac{\beta^2\lambda^4}{k^4\left(k^2+2\lambda^2\right)^2} - \frac{U_T^2\left(2\lambda^2-k^2\right)}{\left(k^2+2\lambda^2\right)}.$$

and

$$\lambda^2 \equiv \frac{f_0^2}{\sigma(\delta p)^2} \qquad (2)$$

Here U_m is the mean (constant) zonal geostrophic velocity, U_T is the mean (constant) thermal wind, λ is the inverse of the Rossby deformation radius for this model, δp is the "thickness" of the layer in Pa (in the two-layer model this is 50000 Pa) and σ is the static stability.

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a) Suppose that β=0, f_0=10^{-4} s^{-1} and $(2\sigma)^{1/2}$≈2×10^{-3} $m^3N^{-1}s^{-1}$. At which wavelengths will the amplitude of the waves grow exponentially?

b) Give two expressions for the dispersion relation if U_T =0. How are these waves called?

c) Demonstrate that, if if U_T =0, standing waves as a response to flow over orography in the quasi-geostrophic two layer model are not possible when the mean flow is easterly (from the east).

Problem 4 (2.5 points)

Structure of a cyclone

Figure 2 show the distribution of absolute vorticity and potential temperature as a function of height and longitude along a constant latitude circle through the centre of a cyclone in the northern hemisphere. The isentropes are indicated by thin black lines (labeled in K; contour-interval is 5 K). Thick lines are isopleths of absolute vorticity (labeled in units of 10^{-4} s^{-1}; contour-interval is 0.5×10^{-4} s^{-1}).

(a) Is this cyclone a tropical cyclone? Why?

(b) In how far can you use quasi-geostrophic theory to describe the dynamics of this cyclone? Why?

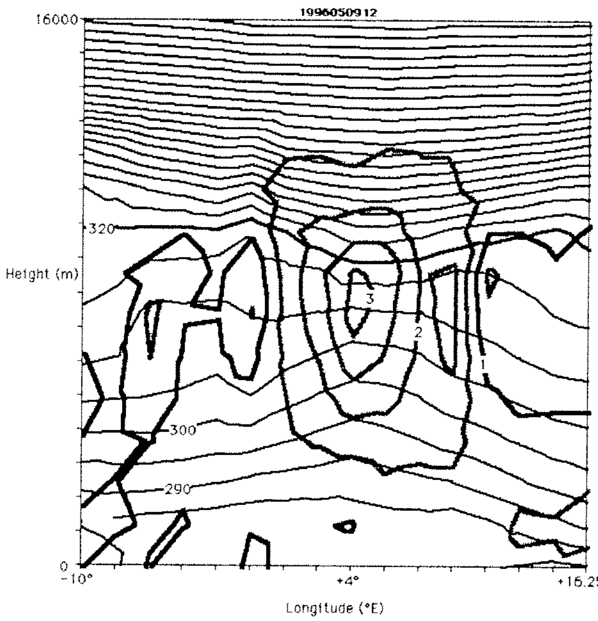

Figure 2a

Problem 5 (2 points)

Q-vector

The frontogenetical function is defiend as follows.

$$\frac{d\left(\vec{\nabla}_h\theta\right)^2}{dt} = 2\vec{\nabla}_h\theta\cdot\frac{d\vec{\nabla}_h\theta}{dt} \approx 2\vec{Q}\cdot\vec{\nabla}_h\theta.$$

Here $\vec{\nabla}_h\theta$ is the horizontal gradient of the potential temperature. Derive an equation for the time-evolution (under adiabatic circumstances) of the x- en y-components of $\vec{Q}$.