M0402 (M)

Exam: Dynamical Meteorology, part 1
Date: November 6, 2009, 0900-1200

In this exam all symbols have their normal definitions.

Answers may be given in either English or Dutch.

Problem 1

Boussinesq approximation

The vertical component of the equation of motion is written as follows.

$$\frac{dw}{dt} = -\theta\frac{\partial \Pi}{\partial z} - g.$$

Apply the Boussinesq approximation to this equation. This implies that you assume that any variable, F, can be expressed as foillows.

$$F(x,y,z,t) = F_0(z) + F'(x,y,z,t).$$

The "basic" time-independent state is assumed to be in hydrostatic balance. Show that the vertical component of the equation of motion can be approximated by

$$\frac{dw}{dt} = -\theta_0\frac{\partial \Pi'}{\partial z} + g\frac{\theta'}{\theta_0},$$

if it is assumed that $F' << F_0$.

Problem 2

Latent heat release

The latent heat, J, that is released in "moist" air (containing water vapour) that is rising is expressed as

$$J = -\frac{L}{m}\frac{dm_v}{dt} \text{ [Joules kg}^{-1}\text{ s}^{-1}\text{]}.$$

Here, m is the mass of the rising air, including the mass of the water vapour, m_v, L is the latent heat of condensation (2.5×10^6 Joules kg^{-1}). Assuming that all condensed water is converted into precipitation, compute the vertically integrated heating due to latent heat release due to condensation of water vapour (in Watts per square metre) if the precipitation intensity is 1 kg m^{-2} per hour.

Problem 3

Hydrostatic balance and scale height

Hydrostatic balance in terms of the Exner function, Π, and the potential temperature, θ, can be expressed as follows.

$$\frac{\partial \Pi}{\partial z} = -\frac{g}{\theta}.$$

Show that Π decreases exponentially with height in a hydrostatically balanced isothermal atmosphere. What is the associated scale height in metres. The relation between Π and θ is

$$\theta = \frac{c_pT}{\Pi},$$

and c_p=1000 J kg^{-1} K^{-1}, g=10 m s^{-2}, T=300 K.

Problem 4

Stability

With the help of the "parcel method", derive an approximate criterion for the stability of hydrostatic balance for an atmosphere with

$$\frac{d\theta_0}{dz} = \text{constant},$$

using the equations given in problem 1.

Problem 5

Multiple choice

(i) Buoyancy waves occur in an statically
- **(a)** unstable atmosphere
- **(b)** stable atmosphere
- **(c)** neutral atmosphere

(ii) In buoyancy waves, the pressure and temperature perturbations are:
- **(a)** in phase
- **(b)** 180° out of phase
- **(c)** 90° out of phase

(iii) If we let a saturated parcel of air ascend, its temperature will
- **(a)** increase
- **(b)** decrease
- **(c)** not change

(iv) CAPE is a measure for
- **(a)** the potential final vertical velocity of air parcels
- **(b)** the maximum possible vertical velocity of air parcels
- **(c)** the layer stability near the capping inversion

(v) The "Middleworld" is defined as
- **(a)** that part of the atmosphere for which the isentropes intersect the tropopause
- **(a)** that part of the atmosphere for which the isentropes intersect the tropopause and the Earth's surface
- **(c)** that part of the atmosphere for which the isentropes span the globe