Exam: Dynamical Meteorology
Date: November, 12, 2010, 09:00-12:00.

In this exam all symbols have their normal definitions.

Answers may be given in either English or Dutch.

Problem 1 (2 points)

Hydrostatic balance and scale height

Hydrostatic balance in terms of the Exner function, Π, and the potential temperature, θ, can be expressed as follows.

$$\frac{\partial \Pi}{\partial z} = -\frac{g}{\theta}.$$

Show that Π decreases exponentially with height in a hydrostatically balanced isothermal atmosphere. What is the associated scale height in metres. The definitions of Π and θ are

$$\theta = T\left(\frac{p_{ref}}{p}\right)^{\kappa};\ \Pi = c_p\left(\frac{p}{p_{ref}}\right)^{\kappa},$$

and c_p=1000 J kg^{-1} K^{-1}, g=10 m s^{-2}, T=300 K.

Problem 2 (2 points)

Vorticity in a "Rankine vortex".

A "Rankine vortex" is an axisymmetric circular vortex with an azimuthal velocity, v_θ, which is a function of the radius, r (the distance from the centre of the vortex), as follows.

$$v_\theta = \frac{v_0 r}{R} \text{ for } r \le R$$

$$v_\theta = \frac{v_0 R}{r} \text{ for } r > R$$

Here v_0 is the maximum wind velocity and R is the radius of maximum wind velocity. Calculate and plot the relative vorticity as a function of r for a Rankine vortex with v_0=40 m s^{-1} and R=40 km.

Problem 3 (2 points)

Q-vector

The frontogenetical function is defined as follows.

$$\frac{\mathrm{d}\left(\vec{\nabla}_{\mathrm{h}}\theta\right)^2}{\mathrm{d}t} = 2\vec{\nabla}_{\mathrm{h}}\theta \cdot \frac{\mathrm{d}\vec{\nabla}_{\mathrm{h}}\theta}{\mathrm{d}t} \equiv 2\vec{Q}\cdot\vec{\nabla}_{\mathrm{h}}\theta.$$

Here $\vec{\nabla}_{\mathrm{h}}\theta$ is the horizontal gradient of the potential temperature. Derive an equation for the x- en y-components of $\vec{Q}$ (under adiabatic circumstances).

Problem 4 (2 points)

Isallobaric wind

Derive equations for the two horizontal components of the isallobaric component of the ageostrophic wind (the *x*-component, u_{isa} and the *y*-component, v_{isa}), on an isobaric surface in terms of

$$\frac{\partial^2 z}{\partial t \partial x} \text{ and } \frac{\partial^2 z}{\partial t \partial y},$$

where z is the height of the isobaric surface. Use the two horizontal components of the momentum equation in isobaric coordinates, which are:

$$\frac{du}{dt} = -g\left(\frac{\partial z}{\partial x}\right)_p + fv;$$
$$\frac{dv}{dt} = -g\left(\frac{\partial z}{\partial y}\right)_p - fu.$$

Problem 5 (2 points)

Multiple choice

(i) The dynamical definition of the tropopause is in terms of
- **(a)** lapse rate
- **(b)** vorticity
- **(c)** potential vorticity

(ii) The energy conservation equation for an air parcel can be written as follows

$$Jdt - c_v dT - pd\alpha = 0$$

The change in the internal energy is given by
- **(a)** the first term
- **(b)** the second term
- **(c)** the third term

(iii) If we let a saturated parcel of air ascend, its equivalent potential temperature will
- **(a)** increase
- **(b)** decrease
- **(c)** not change

(iv) CAPE is a measure of
- **(a)** the potential final vertical velocity of air parcels
- **(b)** the maximum possible change in the vertical velocity of air parcels
- **(c)** the layer stability between the ground and the capping inversion

(v) The equation for thermal wind balance is derived from
- **(a)** the equation for geostrophic balance and the equation for hydrostatic balance
- **(b)** the vorticity equation and the equation for hydrostatic balance
- **(c)** the continuity equation and the equation for geostrophic balance

(vi) A descending air parcel follows
- **(a)** an isentrope on a thermodynamic diagram
- **(b)** a pseudo-adiabat on a thermodynamic diagram
- **(c)** an isotherm on a thermodynamic diagram