Exam: Dynamical Meteorology-1
Date: December 9, 2015, 13:15-16:15

In this exam all symbols have their normal definitions.
Answers may be given in either English or Dutch

Problem 1 (1.25 point)
Lifted Condensation Level

A parcel of air with a temperature of 15°C and a dewpoint temperature of 2°C is lifted from the ground (at 1000 hPa), without mixing with the environment. Determine its lifted condensation level (LCL) and temperature at that level. Will the air parcel reach this level if the potential temperature in the environment at the LCL is 17°C? Assume that the dewpoint lapse rate is 1.8 K/km and that the dry-adiabatic lapse rate is 9.8 K/km.

Problem 2 (1.5 points)
Hydrostatic balance, scale height and geopotential

(a) Show that pressure in an isothermal atmosphere in *hydrostatic balance* decreases exponentially with height. Give an expression for the *scale height*, associated with this exponential decrease.

(b) Show that the geopotential, Φ, at pressure level, p, in an atmosphere in *hydrostatic balance* and with *uniform temperature lapse rate*, Γ, is given by

$$\Phi = \frac{gT_0}{\Gamma}\left[1-\left(\frac{p}{p_0}\right)^{R\Gamma/g}\right],$$

where T_0 and p_0 are the temperature and pressure at sea level.

Note: the vertical component of the equation of motion is

$$\rho\frac{dw}{dt} = -\left(\frac{\partial p}{\partial z}\right)_{x,y,t} - \rho g. \tag{1}$$

Here, w represents the component of the vertical velocity perpendicular to the Earth's surface.

Problem 3 (1.5 points)
Thermal wind balance

The three components of the equation of motion can be expressed as

$$\frac{du}{dt} = -\theta\frac{\partial \Pi}{\partial x} + fv, \tag{2}$$

$$\frac{dv}{dt} = -\theta\frac{\partial \Pi}{\partial y} - fu, \tag{3}$$

$$\theta_0\frac{\partial \Pi_0}{\partial z} = -g \quad . \tag{4}$$

(a) Let us assume a *zonally symmetric* basic state (u_0, θ_0, Π_0) in geostrophic- and hydrostatic balance. Zonally symmetric means that zonal velocity, potential temperature and pressure do not vary in the zonal (x-) direction. The meridional gradient of θ_0 is -10^{-4} K m^{-1}. The vertical gradient of θ_0 is 5×10^{-3} K m^{-1}. Show that in the atmosphere, to a very good degree of approximation,

$$\frac{\partial M_0}{\partial z} \approx -\frac{g}{f\theta_0}\frac{\partial \theta_0}{\partial y}, \tag{5}$$

where $M_0=u_0-fy$ is the linear momentum per unit mass *in the balanced state*.

(b) Show that M ($=u-fy$) is conserved by an air parcel in zonally symmetric conditions.

(c) Show that the meridional slope of isentropes in the balanced zonally symmetric state is equal to $\frac{f\theta_0}{g}\frac{\partial u_0}{\partial z}\left(\frac{\partial \theta_0}{\partial z}\right)^{-1}$

Problem 4 (1.5 points)

Vorticity in a tropical cyclone

Find the *average* relative vorticity in an axisymmetric tropical cyclone, if the tangential velocity is $v=Ar$, for $r<25$ km and $v=B/r$ for $r\geq 25$ km, where r is the distance to the centre of the tropical cyclone, $A=1.6\times10^{-3}$ s^{-1} and $B=10^{6}$ m^{2} s^{-1}. What is the *average* vorticity within the radius of maximum wind? What is the *average* vorticity outside the radius of maximum wind?

Problem 5 (1.5 point)

Vorticity equation in isentropic coordinates

(a) Derive an equation in isentropic coordinates for the time-evolution of isentropic relative vorticity in an *adiabatic, frictionless, hydrostatic atmosphere*. Isentropic relative vorticity is defined as

$$\zeta_\theta = \left(\frac{\partial v}{\partial x}\right)_\theta - \left(\frac{\partial u}{\partial y}\right)_\theta \qquad (6)$$

You may neglecting the terms due the curvature of the coordinate system in the equations of motion:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\left[\frac{\partial u}{\partial x}\right]_\theta + v\left[\frac{\partial u}{\partial y}\right]_\theta + \frac{d\theta}{dt}\frac{\partial u}{\partial \theta} = -\left[\frac{\partial \Psi}{\partial x}\right]_\theta + fv + \frac{uv\tan\phi}{a}, \qquad (7)$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\left[\frac{\partial v}{\partial x}\right]_\theta + v\left[\frac{\partial v}{\partial y}\right]_\theta + \frac{d\theta}{dt}\frac{\partial v}{\partial \theta} = -\left[\frac{\partial \Psi}{\partial y}\right]_\theta - fu + \frac{u^2\tan\phi}{a}. \qquad (8)$$

Here, Ψ is the isentropic streamfunction.

(b) Give a physical interpretation of the terms in this vorticity equation

Problem 6 (1.5 points)

Multiple choice

Indicate the "best" answer

(1) The potential vorticity of a rotating cylinder of air is defined as
  (a) vertically averaged circulation per unit mass
  (b) vertically averaged vorticity per unit height
  (c) vertically averaged vorticity per unit mass

(2) Isentropic density is defined as
  (a) the average density between two isentropic surfaces
  (b) the mass per unit horizontal area between two isentropic surfaces
  (c) the mass per unit horizontal area between two isentropic surfaces which differ in potential temperature difference by 1 °C.

(3) Ideally, the seabreeze, u, at the coast (u is the landward wind component perpendicular to the coast) is described by

$$\frac{\partial^2 u}{\partial t^2} + f^2 u = \frac{A\Omega}{\rho}\sin\Omega t,$$

  where A is a constant. This equation describes
  (a) a forced undamped inertial oscillation with a period equal to $T=2\pi/f$.
  (b) a damped inertial oscillation with a period equal to $2\pi/f$.
  (c) a forced oscillation, consisting of two periods, $T_1=2\pi/f$ and $T_2=2\pi/\Omega$.

(4) Relative humidity of an air parcel is defined as
  (a) the mass density of the water vapour in an air parcel as a fraction of the total mass density of air
  (b) the pressure exerted by water vapour in the air parcel as a fraction of the saturation water vapour pressure at the temperature of the air parcel.
  (c) the mass density of the water vapour in an air parcel as a fraction of the total mass density of water vapour in the air parcel, if it were saturated

(5) The moist adiabatic lapse rate
  (a) decreases with increasing temperature
  (b) is independent of temperature
  (c) increases with increasing temperature

(6) Teten's formula is an integrated form of the Clausius-clapeyron equation,
  (a) which takes into account the dependence of the latent heat of condensation on pressure
  (b) which takes into account the dependence of the latent heat of condensation on temperature
  (c) which takes into account the dependence of the specific gas constant for water water vapour on temperature.

Problem 7 (1.25 points)

Daily wind fluctuations

The figure on the left (below) shows *hourly wind vectors at 10 m above the Earth's surface* as a function of time at six stations in the Netherlands on the very sunny 8 May, 1976. The station on the left (225) is closest to the coast, while the station on the right (290) is furthest from the coast (see the figure on the right for the exact location of the stations). Give an interpretation of these wind observations.