Exam: Dynamical Meteorology, part 2
Date: January 27, 2016, 13:30-16:30

Give the "best" answer
Symbols have their usual meaning
Be tidy

Problem 1 (1.6 point)

Thermal and dynamic structure of a cyclone

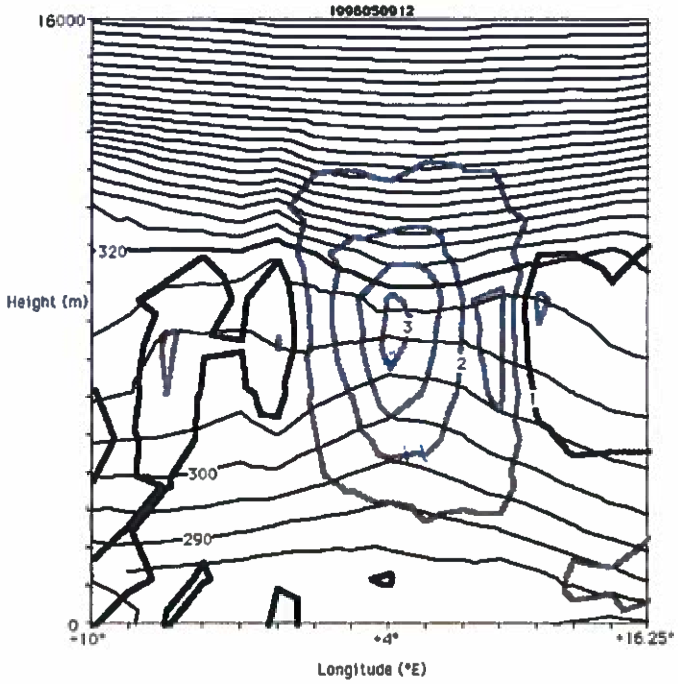

Figure 1 (explanation below)

Figure 1 shows a vertical cross section of the atmosphere, in terms of absolute vorticity and potential temperature as a function of height and longitude, at a latitude equal to 52.25°N through the centre of a cyclone. Isentropes are indicated by thin black lines (labeled in K; contour-interval is 5 K). Thicker lines are isopleths of absolute vorticity (labeled in units of 10^{-4} s^{-1}; contour-interval is 0.5×10^{-4} s^{-1}). The thick black line corresponds to the $+10^{-4}$ s^{-1}-isopleth of absolute vorticity. The existence of the cyclone can be explained by the presence of a positive potential vorticity anomaly and the assumption that the atmosphere is in a state of thermal wind balance.

(a) Based on the information given in **figure 1**, at which height and which longitude, approximately, is the centre of this positive potential vorticity anomaly located?

(b) In which way is the thermal and dynamic structure of this cyclone in accord with the predictions made by the solution of the potential vorticity inversion equation for an axisymmetric potential vorticity anomaly?

(c) Why can you, strictly speaking, *not* apply quasi-geostrophic theory to describe the structure and dynamics of the core of this cyclone?

Problem 2 (1.8 points)

Q-vector

The frontogenetical function is defined as follows.

$$\frac{d\left(\vec{\nabla}_h\theta\right)^2}{dt} = 2\vec{\nabla}_h\theta\cdot\frac{d\vec{\nabla}_h\theta}{dt} = 2\vec{Q}\cdot\vec{\nabla}_h\theta.$$

Here $\vec{\nabla}_h\theta$ is the horizontal gradient of the potential temperature.

(a) Derive equations for the *x*- en *y*-components of $\vec{Q}$, under adiabatic circumstances.

(b) Identify the terms in these equations that represent the mechanism responsible for changing the direction of the horizontal temperature gradient.

Problem 3 (2.1 points)

Baroclinic waves

The dispersion relation for waves of the form $A\exp[ik(x-ct)]$ in the two-layer quasi-geostrophic model is

$$c = U_m - \frac{\beta(k^2+\lambda^2)}{k^2(k^2+2\lambda^2)} \pm \delta^{1/2}, \qquad (1)$$

with

$$\delta = \frac{\beta^2\lambda^4}{k^4(k^2+2\lambda^2)^2} - \frac{U_T^2(2\lambda^2-k^2)}{(k^2+2\lambda^2)}$$

and

$$\lambda^2 = \frac{f_0^2}{\sigma(\delta p)^2} \qquad (2)$$

Here U_m is the mean (constant) zonal geostrophic velocity, U_T is the mean (constant) thermal wind, λ is the inverse of the Rossby deformation radius for this model, δp is the "thickness" of the layer in Pa (in the two-layer model this is 50000 Pa) and σ is the static stability.

(a) Suppose that β=0, $f_0 = 10^{-4}$ s^{-1} and $\sigma = 2\times10^{-6}$ m^{2}Pa^{-2}s^{-2}. At which wavelengths will the amplitude of the waves grow exponentially?

(b) Give two expressions for the dispersion relation if U_T =0. How are these waves called?

(c) Demonstrate that, if U_T =0, standing waves as a response to flow over orography in the quasi-geostrophic two layer model are not possible when the mean flow is westward.

Problem 4 (2.4 points)

The structure of planetary waves

Let us suppose that distribution of geopotential, Φ (=gz), in an atmosphere, which is in hydrostatic balance ($\partial p/\partial z = -\rho g$) and in approximate geostrophic balance, is given by

$$\Phi(x,y,p,t) = \Phi_0(p) + \left\{\frac{f_0V_0}{k}\sin(kx-ct) - f_0U_0(y-y_0)\right\}\cos\left(\frac{\pi p}{2p_0}\right) \qquad (3)$$

Here, $k=2\pi/L_x$, c is the zonal component of the phase velocity, V_0 and U_0 are constant velocities, p_0 is the constant pressure at the earth's surface, y_0 is the value of y corresponding to the reference latitude where $f=f_0$ and Φ_0 is a function of pressure. Geostrophic balance implies that $fu=-\partial\Phi/\partial y$ and $fv=\partial\Phi/\partial x$. Apply the quasi-geostrophic approximation.

(a) Derive a mathematical expressions for both components of the geostrophic wind as function x, y and p in the vicinity of the reference latitude.

(b) Derive a mathematical expression for the temperature as function x, y and p. Use the ideal gas law, $p=\rho RT$.

(c) At which pressure is the baroclinicity of the atmosphere largest, i.e. at which level will the atmosphere most likely exhibit baroclinic instability?

(d) The meridional heat flux is proportional to $[v^*T^*]$, where the square brackets denote the zonal average and the asterisk denotes the deviation from the zonal average. Is the meridional heat transport poleward, equatorward, or zero?

Problem 5 (2.1 points)
Multiple choice
Indicate the "best" answer

1. Through the excitation of buoyancy waves the Earth exerts a drag on the atmosphere if the fluctuations of the velocity components in the atmosphere, associated with these waves, obey the following relation at any level in the atmosphere:

(a) $\partial \overline{u'w'}/\partial x > 0$
(b) $\partial \overline{u'w'}/\partial z > 0$
(c) $\overline{u'w'} < 0$

2. A rule that can be deduced from the solution of the quasi-geostrophic omega equation,

$$\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) + \frac{f_0^2}{\sigma}\frac{\partial^2 \omega}{\partial p^2} = -\frac{2R}{p\sigma}\vec{\nabla}\cdot\vec{Q}_g,$$

with the static stability parameter,

$\sigma > 0,$

is the following:

(a) upward motion is found in regions where the Q_g-vector diverges
(b) downward motion is found in regions where the Q_g-vector diverges
(c) downward motion is found in regions where the Q_g-vector converges

3. "Lamb" waves are

(a) horizontally propagating sound waves with no vertical motion
(b) vertically propagating sound waves with no vertical motion
(c) vertically propagating sound waves

4. The equation,

$$-\frac{g}{c_0^2}w + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{J}{\Pi_0\theta_0},$$

where c_0 represents the speed of sound waves in the reference state, represents the

(a) continuity equation after applying the "shallow Boussinesq approximation"
(b) continuity equation after applying the "deep Boussinesq approximation"
(c) adiabatic continuity equation

5. The aspect ratio (horizontal scale divided by vertical scale) of quasi-geostrophic circulation systems is in the order of,

(a) N/F
(b) F/N
(c) N/f_0

where N is the Brunt-Vaisala frequency, F is the inertial frequency and f_0 is the Coriolis parameter at a reference latitude in the mid-latitudes.

6. In a stationary, steady state anticyclone, which is not intensifying or weakening in the northern hemisphere the isollabaric wind,

(a) blows in southward direction
(b) is non-existent
(c) blows in northward direction

7. If there is warm advection in the northern hemisphere,

(a) the geostrophic wind turns clockwise with increasing height
(b) the geostrophic wind decreases in absolute value with increasing height
(c) the geostrophic wind turns anti-clockwise with increasing height