Test examination ICE AND CLIMATE

1. We consider the effect of katabatic flow on the energy budget of a melting glacier surface.

a. Give a qualitative description of katabatic flow over mountain glaciers (dynamics, first-order momentum and heat budget).

A simplified expression for the energy balance M at a melting glacier surface can be written as (fluxes towards the surface are positive):

$$M = (1-\alpha)Q - \varepsilon_s \sigma T_s^4 + \varepsilon_a \sigma T_a^4 + k(T_a - T_s)^2$$

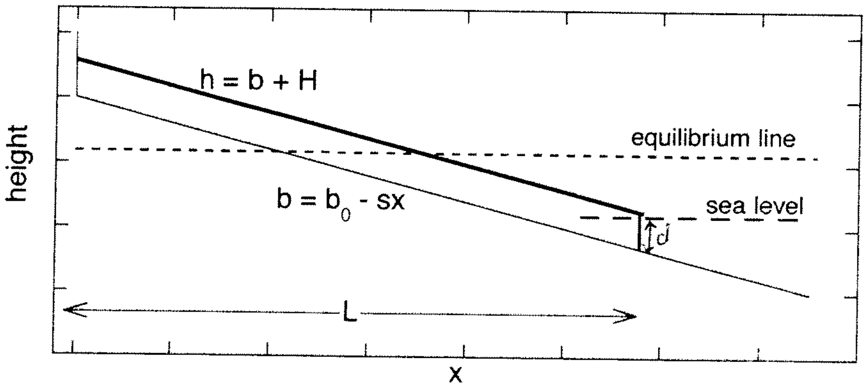
In this expression α is the albedo, Q the incoming solar radiation, T_s the surface temperature, T_a the air temperature at a certain height, k a turbulent exchange coefficient, σ the Stefan-Boltzmann constant ($5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}). The effective emissivities of the surface and the atmosphere are denoted by ε_s and ε_a, respectively. The last term in the equation describes the turbulent sensible heat flux due to the presence of katabatic flow.

b. On a sunny day in summer with significant melting we have $T_a > T_s$. In spite of this, the net longwave balance is often negative. Can you explain this?

c. Investigate the sensitivity of M to a change in air temperature (T_a'). Assume that $T_a' << (T_a - T_s)$.

d. On a warm summer day we have $T_a = 285$ K. If the air temperature now increases, both the net longwave balance and the turbulent heat flux increase. Which process is more important? [model parameters: $\varepsilon_a = 0.7$, $k = 0.5$ K^{-2} W m^{-2}]

2. We consider a simple model for a calving glacier of constant width, having a constant thickness H and flowing on a bed $b(x)$ with a (small) constant slope s: $b = b_o - sx$. Water and ice density are denoted by ρ_w and ρ_i, respectively.



a. The maximum possible glacier length is $L_{\max}$. Find an expression for $L_{\max}$.

The surface balance rate is written as $\dot{b} = \beta(h - E)$. Here β is the balance gradient and E the equilibrium-line altitude. The calving rate at the glacier front is proportional to the water depth (constant of proportionality c).

b. Solve for the equilibrium solutions of L as a function of E.

c. Make a qualitative sketch of the solution $L(E)$.

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3. We consider the depression of the earth's crust due to an ice load.

a. Derive the profile $h(r)$ of a perfectly plastic ice sheet with radius R on a flat bed [denote yield stress by τ_0, ice density by ρ_i, gravitational acceleration by g]. What is the expression for the maximum ice thickness $H_{\max}$?

Now assume that the earth's crust is depressed by the ice load by an amount d in the centre of the ice sheet. The mantle density is ρ_m.

b. Find an expression for d for the case that the lithosphere has no strength.

It is more realistic to assume that the lithosphere has a certain strength.

c. What is the effect of this? Make a sketch of how the quantity $(d/H_{\max})$ varies with R.

4. The thermodynamic equation for solid ice reads:

$$\frac{\partial T}{\partial t} = -v_i \frac{\partial T}{\partial x_i} + \frac{\partial}{\partial x_i}\left(K \frac{\partial T}{\partial x_i}\right) + \frac{\dot{\varepsilon}_{ij}\,\tau_{ij}}{J\rho c_p} + \frac{L_m}{\rho c_p}\dot{m}$$

a. Simplify the equation for application at an ice dome on the Antarctic ice sheet (all horizontal gradients vanish, no melting or refreezing). Which processes determine the vertical temperature profile in this case?

b. During the last ice age the surface temperature on Antarctica was considerably lower and the accumulation rate smaller. Discuss the implications for the basal ice temperature.

5. H_2O with the stable oxygen isotope ^{18}O is present in all forms of water on the earth. The relative amount of ^{18}O varies and this fact has been used extensively in the study of palaeoclimates.

a. Variations in ^{18}O are normally expressed in a quantity $\delta^{18}O$. How is this quantity defined?

b. What processes are the cause for fluctuations in $\delta^{18}O$ as measured in deep-sea sediments and ice cores?