1.

a

Extra-terrestrial irradiance is the solar incoming radiation on a horizontal surface. The following processes lead to a reduction in the incoming radiation measured at the glacier's surface:

- Interaction in the atmosphere (absorption and scattering by molecules, reflection by clouds)
- Shading by surrounding terrain
- Inclination of the glacier's surface
- Reflection at the glacier's surface (bare ice has an albedo of ≈ 0.5 whereas the albedo of snow is even higher)

b

Mechanism of katabatic wind
The 'glacier wind', also known as the katabatic wind, is forced by a potential temperature deficit (θ) in the layer just above the glacier's surface. This deficit in temperature is defined as the difference between the actual potential temperature of the air and some background potential temperature (extrapolated with a constant lapse rate). The colder air has a higher density than its surrounding air, and the buoyancy force associated with this density difference drives the cold air downslope.

Influence of katabatic wind on the glacier's mass balance
The mass balance of a glacier is closely coupled to the surface energy balance: the latter determines how much energy is available for melting at the glacier's surface. A melting glacier surface on a summer day is often accompanied by a katabatic wind, that effectively 'pumps' heat from the warmer atmosphere to the surface. The turbulent heat flux accounts for this energy flux in the energy balance. A simple parameterization of the turbulent heat flux is:

$$H_s = \rho \, c_p \, C^* \, (T_a - T_s) \tag{1}$$

C^* is the turbulent exchange coefficient. An extra term due to katabtic wind can be added to this coefficient. Another effect of the katabatic wind on the mass balance can be through snow drift, which can either be a positive or negative term in the mass balance, depending on acceleration or deceleration of the katabatic flow.

2.

a

The end of the glacier is at the point where the hydrostatic pressure of water and ice at the bed are equal (→ grounding line). The water depth at this location is given by:

$$\underbrace{\rho_i\, g\, H_i}_{p_i} = \underbrace{\rho_w\, g\, H_w}_{p_w} \tag{1}$$

$$H_w = H_i \left(\frac{\rho_i}{\rho_w}\right) \tag{2}$$

Assuming that sea level is at the elevation where the vertical coordinate is 0, the water depth can be set equal to the glacier's bed equation (multiplied by (-1)):

$$s\, L_{max} - b_0 = H_i \left(\frac{\rho_i}{\rho_w}\right) \tag{3}$$

Solving the above equation for L_{max} yields:

$$L_{max} = \frac{b_0 + H\,(\rho_i/\rho_w)}{s} \tag{4}$$

b

In a steady state (or equilibrium), the mass input of a glacier over the surface is equal to the mass output by calving. Hence

$$B_s = C \tag{5}$$

where B_s is the total surface balance and C the mass loss by calving. Using the equations $(b = b_0 - s\,x)$, $(h = b + H)$ and $(\dot{b} = \beta\,(h - E))$, the total surface balance can be calculated as:

$$B_s = \beta\, W \int_0^L (b_0 - s\,x + H - E)\, dx \tag{6}$$

$$= \beta\, W \left[L\,(b_0 + H - E) - \frac{1}{2}\, s\, L^2\right] \tag{7}$$

The mass loss by calving is

$$C = c\, W\, H\,(s\, L - b_0) \tag{8}$$

where it is necessary to include W and H because the total horizontal volume flux of ice at the glacier's terminus is proportional to W and H. To find the equilibrium length, we look at two different cases:

Land terminated $(L \leq b_0/s)$
For this case, C is 0 and therefore equation (7) can also be set to zero. Solving this equation for L yields:

$$L = \frac{2\,(b_0 + H - E)}{s} \tag{9}$$

Water terminated $(L > b_0/s)$
For this case, both equations (7) and (8) have to be plugged into equation (5):

$$\beta\, W\,\left[L\,(b_0 + H - E) - \frac{1}{2}\,s\,L^2\right] - c\,W\,H\,(s\,L - b_0) = 0 \tag{10}$$

Rearranging the above equation (and dividing by $-W$) yields:

$$\left[\frac{\beta\,s}{2}\right]\,L^2 + \left[-\,\beta\,(b_0 + H - E) + c\,H\,s\right]L + \left[-\,c\,H\,b_0\right] = 0 \tag{11}$$

As we are interested to find a function $L(E)$, we can rearrange the above equation yet again to isolate E (all other variables are constant):

$$\left[\frac{\beta\,s}{2}\right]\,L^2 + \left[(\beta)\,E + (-\beta\,b_0 - \beta\,H + c\,H\,s)\right]L + \left[-\,c\,H\,b_0\right] = 0 \tag{12}$$

Combining all constants

$$\lambda = \frac{\beta\,s}{2} \tag{13}$$

$$\mu = -\,\beta\,b_0 - \beta\,H + c\,H\,s \tag{14}$$

$$\nu = -\,c\,H\,b_0 \tag{15}$$

and using the ABC-formula yields:

$$L = \frac{-\beta\,E - \mu \pm \sqrt{(\beta\,E + \mu)^2 - 4\,\lambda\,\nu}}{2\,\lambda} \tag{16}$$

c

Producing a sketch of the equilibrium length equation for the water terminated glacier is complicated. However, we can start by making a sketch of equation (9) and (4) and drawing the boundary line between the land and water terminated glacier:

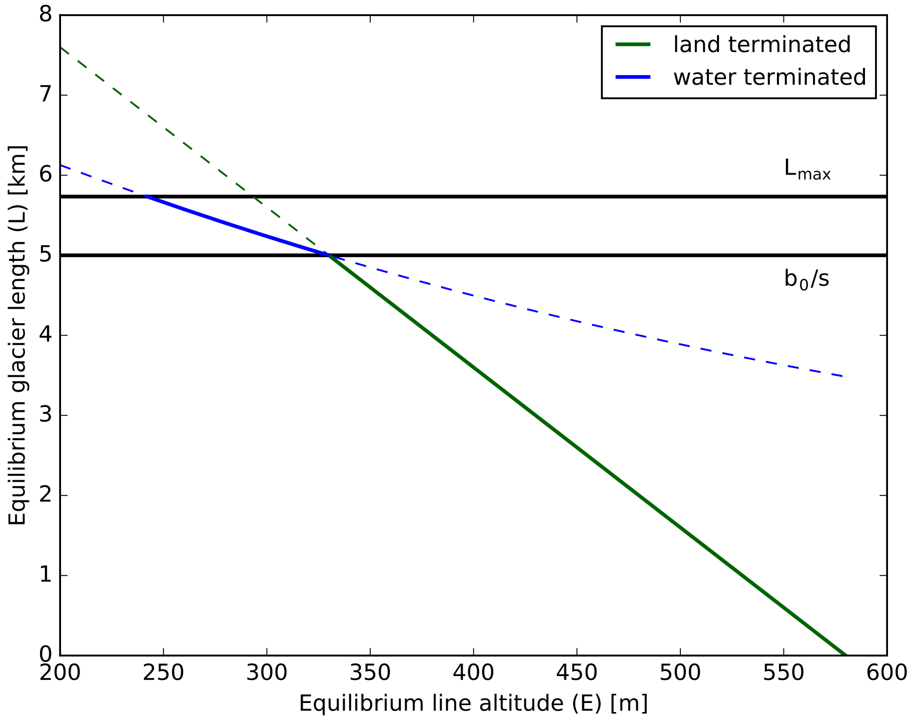

Figure 1: Equilibrium glacier length calculated with the following numerical values: $b_0 = 500\,m$, $s = 0.1$, $H = 80\,m$, $\beta = 1\,a^{-1}$, $c = 50\,a^{-1}$

Considering this plot, it is now obvious how the equilibrium length function $E(L)$ for the water terminated glacier approximately looks like.

3.

a

The force balance equation of an ice sheet - shelf system is:

$$\underbrace{\rho_i\, g\, H \frac{\partial h}{\partial x}}_{I} + \underbrace{\frac{\partial}{\partial x}\left(2H\overline{\tau'_{xx}}\right)}_{II} + \underbrace{\tau_b}_{III} = 0 \tag{1}$$

The terms in the above equation are:

I The horizontal pressure gradient, also often called the driving force. It consists of the product of density (ρ), acceleration of gravity (g), the ice thickness (H) and the surface slope (dh/dx). The driving stress is balanced by the other terms that usually give resistance to the flow.

II The horizontal gradient of the normal (or longitudinal) deviatoric stress. The overbar indicates a vertically averaged quantity. This term describes stretching or compression of the ice flow.

III The basal drag which is the friction exerted to the flow by the bedrock.

b

The basal drag (τ_b) can be neglected for an ice shelf, hence equation (1) reduces to:

$$\frac{\partial}{\partial x}\left(2H\overline{\tau'_{xx}}\right) = -\rho_i\, g\, H \frac{\partial h}{\partial x} \tag{2}$$

c

The surface elevation (h) of a floating ice shelf can be related to the ice thickness (H) by

$$h = \left(1 - \frac{\rho_i}{\rho_w}\right) H \tag{3}$$

Computing dh/dH of the above equation, solving for dh and plugging this in equation (2) yields:

$$\frac{\partial}{\partial x}\left(2H\overline{\tau'_{xx}}\right) = -\rho_i\, g\, H \left(1 - \frac{\rho_i}{\rho_w}\right) \frac{\partial H}{\partial x} \tag{4}$$

Integrating equation (4) with respect to x yields

$$2H\overline{\tau'_{xx}} = -\frac{1}{2}\rho_i\, g \left(1 - \frac{\rho_i}{\rho_w}\right) H^2 \tag{5}$$

which can be rearranged to:

$$\overline{\tau'_{xx}} = -\frac{1}{4}\rho_i\, g \left(1 - \frac{\rho_i}{\rho_w}\right) H \tag{6}$$

This shows that the longitudinal stress deviator attains a minimum value where H is small. An ice shelf has its maximum ice thickness at the grounding line. From there it thins, with minimum thickness at the calving front, where the minimum value of $\overline{\tau'_{xx}}$ is reached.

d

The depth of crevasses in ice shelves is limited by the balance between a large-scale normal stress gradient (tending to open the crevasses) and a small-scale plastic deformation (tending to close the crevasses). However, when melt water accumulates in crevasses, they can become deeper and in the end destabilise the ice shelf. A series of warm summers probably lead to the break-up of large parts of the Larsen ice shelf.

4.

a

The stress balance at the base of the ice sheet can be approximated by

$$\rho_i \, g \, H \left| \frac{dh}{dx} \right| = \tau_0 \tag{1}$$

where τ_0 is called the yield stress. By neglecting isostasy ($h = H$) and only looking at the ice sheet from the edge to the centre ($dH \,/\, dx > 0$), the above equation can be written as:

$$H \, dH = \frac{\tau_0}{\rho_i \, g} \, dx \tag{2}$$

Integration yields:

$$\frac{H^2}{2} = \frac{\tau_0}{\rho_i \, g} \, x + C \tag{3}$$

Using the boundary condition $H(x = 0) = 0$, the above equation can be written as:

$$H(x) = \sqrt{\frac{2 \, \tau_0}{\rho_i \, g} \, x} \tag{4}$$

b

Combining equation (2) from the previous exercise with the given equation for τ_0:

$$H \, dH = \left(\frac{\tilde{\tau}_0}{\rho_i \, g} + \frac{\tau_1}{\rho_i \, g \, L} \, x \right) \, dx \tag{5}$$

Integration yields:

$$\frac{H^2}{2} = \frac{\tilde{\tau}_0}{\rho_i \, g} \, x + \frac{\tau_1}{2 \, \rho_i \, g \, L} \, x^2 + C \tag{6}$$

Using the boundary condition $H(x = 0) = 0$, the above equation can be written as:

$$H(x) = \sqrt{\frac{2 \, \tilde{\tau}_0}{\rho_i \, g} \, x + \frac{\tau_1}{\rho_i \, g \, L} \, x^2} \tag{7}$$

To compare the sliding with the non-sliding case, we first have to modify the provided linear function for the yield stress. A reasonable assumption is that sliding goes to 0 at the centre of the ice sheet ($x = L$). Hence, for this location, the provided linear function for the yield stress should return the constant yield stress used in exercise **a**. Using $\tau_1 = \tau_0 \,/\, 2$, we can rewrite the linear function for the yield stress for this location as:

$$\tau_0 = \tilde{\tau}_0 + \frac{\tau_0}{2} \tag{8}$$

Solving the above equation for $\tilde{\tau}_0$ yields $\tilde{\tau}_0 = \tau_0 \,/\, 2$. Plugging this equation and $\tau_1 = \tau_0 \,/\, 2$ in equation (7) and dividing it by the $H(x)$ computed in exercise **a** yields:

$$\frac{H_b}{H_a} = \sqrt{\frac{\frac{\tau_0}{\rho_i \, g} \, x + \frac{\tau_0}{2 \, \rho_i \, g \, L} \, x^2}{\frac{2 \, \tau_0}{\rho_i \, g} \, x}} = \sqrt{\frac{1}{2} + \frac{x}{4 \, L}} \tag{9}$$

c

In case of perfect isostatic equilibrium, the ice thickness (H) can be expressed in terms of surface elevation (h) as:

$$H = (1 + \zeta)\, h \tag{10}$$
$$\zeta = \frac{\rho_i}{\rho_m - \rho_i} \tag{11}$$

Plugging the above relation and the given equation for τ_0 in equation (1) yields:

$$\rho_i\, g\, (1 + \zeta)\, h \left|\frac{dh}{dx}\right| = \tilde{\tau}_0 + \frac{x}{L}\, \tau_1 \tag{12}$$

Again, by only looking at the ice sheet from the edge to the centre ($dh\, /\, dx > 0$), the above equation can be rewritten to:

$$\rho_i\, g\, (1 + \zeta) \int h\, dh = \int \left(\tilde{\tau}_0 + \frac{x}{L}\, \tau_1\right) dx \tag{13}$$

Integration on both sides yields:

$$\rho_i\, g\, (1 + \zeta)\, \frac{h^2}{2} = \tilde{\tau}_0\, x + \frac{x^2}{2\, L}\, \tau_1 + C \tag{14}$$

Applying the same boundary condition as above shows once again that $C = 0$. Solving the above equation for h yields:

$$h(x) = \sqrt{\frac{2\, \tilde{\tau}_0\, x}{\rho_i\, g\, (1 + \zeta)} + \frac{x^2\, \tau_1}{L\, \rho_i\, g\, (1 + \zeta)}} \tag{15}$$

And the surface elevation at the centre ($x = L$) is:

$$h_{max} = \sqrt{\frac{2\, \tilde{\tau}_0\, L}{\rho_i\, g\, (1 + \zeta)} + \frac{L\, \tau_1}{\rho_i\, g\, (1 + \zeta)}} \tag{16}$$

5.

a

Ice cores and marine cores have the advantage that they contain a near-continuous record of accumulated snow or marine sediments (as opposed to terrestrial records that often only cover a certain distinct period). Both marine and ice core records offer the possibility to cover a time scale of $> 10^5$ a (marine records even $> 10^6$ a), ending at present-day. Compared to terrestrial archives, ice core have also the advantages that they provide a high temporal resolution (depending on the surface accumulation history).

b

Information stored in ice cores

- Ice matrix
  - $\delta^{18}O$ → temperature proxy
  - History of local accumulation rate (if age is known)
  - Orientation and size of ice crystals → deformation history
- Enclosed air (→ age difference to ice matrix)
  - Past atmospheric composition → greenhouse gas concentrations (CO_2, CH_4, etc.)
  - Isotopic composition of trace gasses
- Other components:
  - Dust → atmospheric dust concentrations
  - Volcanic ash → (large) volcanic eruptions, layer dating
  - Radioactive material → layer dating
  - Biological substances: pollen, DNA, etc.

$\delta^{18}O$ method (ratio of ^{18}O to ^{16}O)

- Ice cores
  - Isotopic fractionation (lower abundance of the heavy isotope in the vapour phase)
  - Depletion of ^{18}O in air parcel → relation to temperature during condensation
- Deep-sea cores
  - Abundance of ^{18}O stable isotope in the skeleton of foraminifera
  - Sea water temperature → diffusion of oxygen through membrane of micro-organism temperature dependent (→ influences isotope ratio)
  - Changing land ice volume (isotopic fractionation during evaporation from ocean surface)