EXAM OCEAN WAVES

10 June 2011, 11.00 - 14.00 hours

Four problems; all items have equal weight

Remark 1: answers may be written in English or Dutch

Remark 2: in all questions you may use $g = 10$ ms^{-2}, $\rho = 10^3$ kgm^{-3} and $\tau = 0.1$ Nm^{-1}.

Problem 1

The equations of motion for linear water waves read

$$\begin{aligned}
&\nabla^2\phi = 0 && \\
&\frac{\partial \phi}{\partial z} = 0 && \text{at} \quad z = -H, \\
&\frac{\partial \phi}{\partial z} = \frac{\partial \zeta}{\partial t} && \text{at} \quad z = 0, \\
&\frac{\partial \phi}{\partial t} + g\zeta + \frac{p_0}{\rho} - \frac{\tau}{\rho}\nabla_h^2\zeta = 0 && \text{at} \quad z = 0.
\end{aligned} \tag{1}$$

a. Give a physical interpretation of the first boundary condition at $z = 0$ in system (1). Provide your explanation with a clear situation sketch.

b. Show that a free wave of the system presented above is given by

$$\phi = A\,c\,\cosh[\kappa(z+H)]\,\sin[\kappa(x-ct)]\,,$$

where the phase velocity obeys

$$c^2 = \left[\frac{g}{\kappa} + \frac{\tau\kappa}{\rho}\right]\tanh(\kappa H)\,,\,.$$

Also, express variable A in terms of the amplitude a of the free surface elevations.

Consider waves, described by the equations of motion presented above, with an amplitude of 0.5 m and with a wavelength of 10π m in water with an undisturbed depth of 10 m.

c. Compute the net mass transport that is induced by these waves.

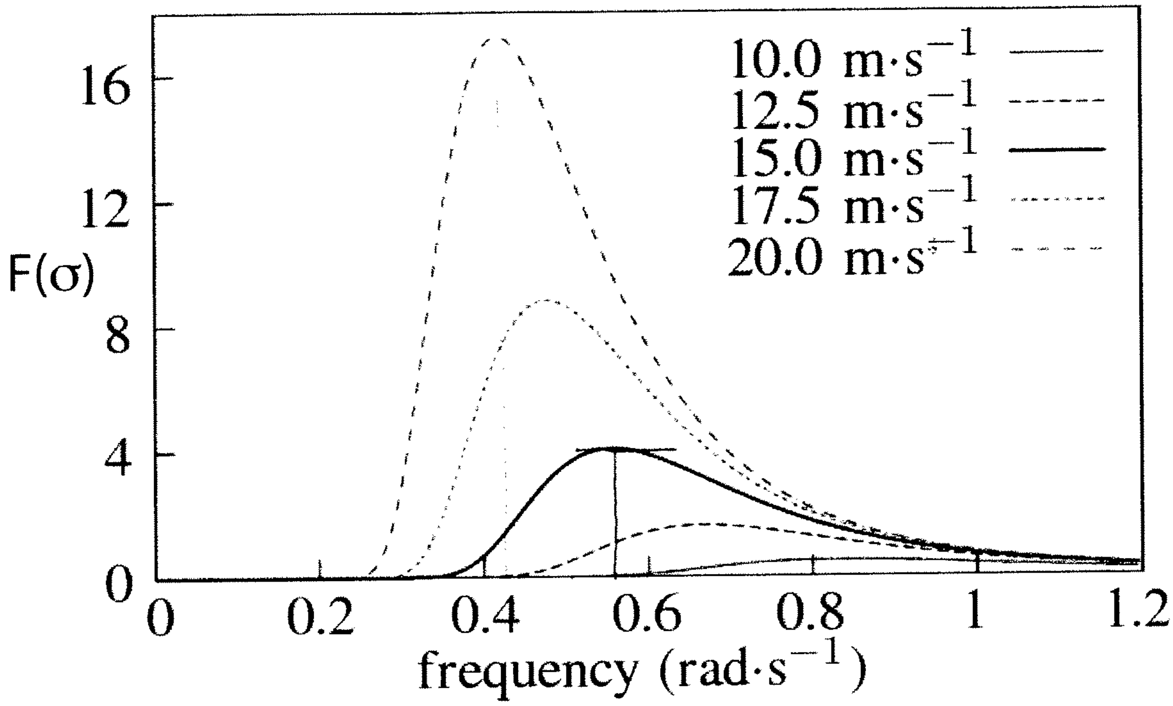
d. Compute the amplitude of the particle orbits at the bottom.

Problem 2

During a measuring campaign, sea surface variations are recorded at a number of locations in the Atlantic Ocean. After the campaign, frequency spectra are calculated from the time series. The calculated spectra are fitted to the Pierson-Moskowitz spectrum

$$F(\sigma) = \alpha\, g^2\, \sigma^{-5}\, \exp\left[-\frac{5}{4}\left(\frac{\sigma}{\sigma_p}\right)^{-4}\right].$$

This spectrum is sketched below for different wind speeds.



a. Name parameter σ_p and estimate its value for a wind speed of 15 ms^{-1}.

b. Sketch in one figure the Pierson-Moskowitz spectrum and the JONSWAP spectrum for a wind speed of 20 ms^{-1} and indicate the differences.

c. One spectrum turns out to be characterised by the parameter values $\alpha = 0.01$ and σ_p=0.5 s^{-1}. From this, calculate the rootmean-square wave height H_{rms}, defined as

$$H_{rms} = \left[8 < \zeta^2 >\right]^{1/2}$$

Hint: $\frac{d}{dx}\left[\exp(x^\mu)\right] = \mu\, x^{\mu-1} \exp(x^\mu)$.

d. Discuss the main aspects of the Phillips mechanism that describes the initial growth of the sea waves.
Make a clear sketch to illustrate your arguments.
Limit your answer to 1 A4 page maximum.

Question 3

On 19 March 2011 (close to beginning of spring) the unusual situation occurred that it was full moon and the moon was also close to its perigee (distance earth-moon is minimum).

a. Was it spring tide, neap tide or none of these on this date?
   Motivate your answer and include a clear situation sketch.

b. Were there strong declination tides on this date?
   Again, explain your answer and include a sketch.

c. Suppose that at a certain location the typical tidal range at the day of this event was 3 m. When one would measure the tidal range during the following new moon, would you expect a larger value, a smaller value, the same value or is it not possible to answer this question?
   Again, explain your answer and include a sketch.

Problem 4

Consider a frictionless fluid in an open channel (width B) with constant water depth H on the rotating earth. Assume that the dynamics of the fluid is governed by the linearised depth-averaged shallow water equations on the f-plane. Choose the x-axis in the along-channel direction.

a. Which free travelling waves are described by this model (no derivation asked)? Specify and sketch the dispersion relations and discuss the physical characteristics of these waves. Pay specific attention to the velocity field and the role of the Rossby deformation radius.

b. Suppose that the Pacific Ocean is schematised as a rectangular channel with width $B = 1 \times 10^7$ m and depth $H = 4 \times 10^3$ m at latitude $\varphi = 45°$. Assume $f = 10^{-4}$ s^{-1}. Which waves, as discussed in item a, can be excited by the semi-diurnal lunar tide? Explain your answer.

c. Assume that the channel is not open, but that a coast is present at location $x = 0$. Explain that this coast will result in the excitation of a new type of waves. Name these waves, specify their dispersion relation and discuss their characteristics.

d. Consider again the model of the Pacific of item b, but now schematised as a half-open basin.
Compute the characteristic length scale of the waves of item c that have a frequency equal to that of the semi-diurnal lunar tide.

END