

FINAL EXAM OCEAN WAVES

7 November 2013, 13.30-15.30 hours (2 hours)

Three problems; all items have equal weight

Remark 1: Answers may be written in English or Dutch.

Remark 2: Given: $g = 10 \text{ ms}^{-2}$, $\rho = 10^3 \text{ kgm}^{-3}$, $\Omega = 7.3 \times 10^{-5} \text{ s}^{-1}$, $\tau = 0.1 \text{ Nm}^{-1}$.

Problem 1

During a measuring campaign in the North Sea, wave frequency spectra have been determined from measured sea surface variations. The results are shown in the figure below.

The measured waves are in deep water.

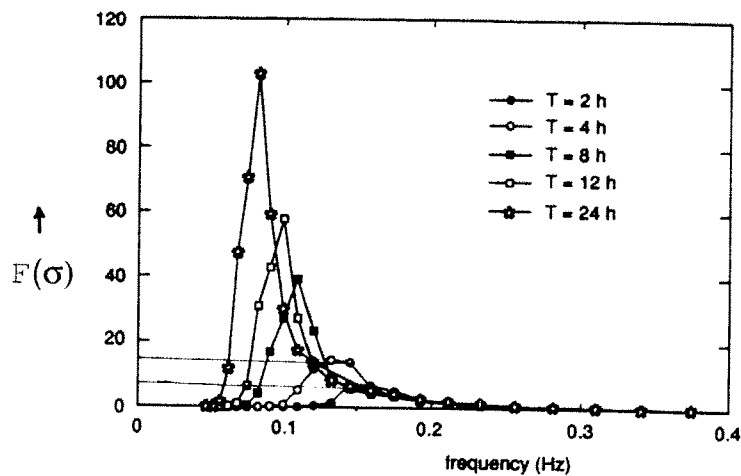


Figure 1: Measured wave spectra at different times T at a location in the North Sea.

The calculated spectra are well described by the JONSWAP spectrum

$$F(\sigma) = \alpha g^2 \sigma^{-5} \exp \left[-\frac{5}{4} \left(\frac{\sigma}{\sigma_p} \right)^{-4} \right] F_3(\sigma).$$

- Discuss how, and under what conditions, the wave spectra at the indicated times are calculated from measured sea surface elevations. No derivations asked.
- Sketch the behaviour of function $F_3(\sigma)$ as a function of σ . Explain its shape and what it accounts for.

- Assume that for $\sigma > 2\sigma_p$ the wave spectrum reads

$$F(\sigma) \simeq \alpha g^2 \sigma^{-5} \quad \text{for } \sigma > 2\sigma_p.$$

Derive an explicit expression for the energy density that is contained in this high-frequency part of the spectrum.

- Name and discuss the physical mechanism that is responsible for the shift of the peak frequency towards lower values with increasing time.

Remark: focus on physical aspects, supply your answer with clear sketches and limit your answer to at most 0.5 page A4.

$$\frac{\rho g}{m^3} \frac{m^2}{s^4} s^4 \frac{1}{s^2} = \frac{\rho g}{s^2}$$

$$-\frac{1}{4} \alpha g^2 \sigma^{-4}$$

$$\frac{m^2}{s^4} s^2$$

$$\frac{m^2}{m^3} \frac{1}{s^2}$$

Problem 2

On 19 March 2013 (almost the beginning of spring) the moon was in its first quarter, and the moon was also close to its apogee position (distance earth-moon is maximum). At latitude 52° N the moon had, at its highest position in the sky, an angle of 32° with respect to the vertical axis at the earth's surface.



- a. Was it spring tide, neap tide or none of these on this date?

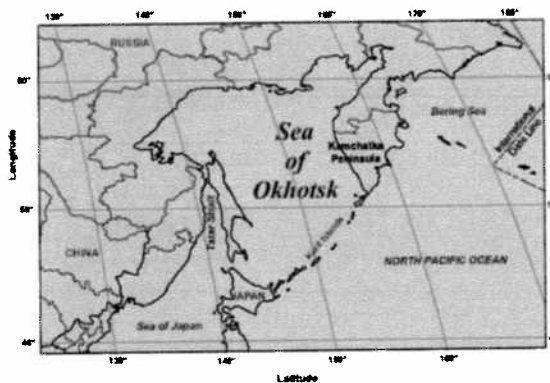
Motivate your answer and include a clear situation sketch, in which you show the tidal forces that on this date acted on the surface of the earth due to moon and sun.

- b. Argue whether on this date strong declination tides occurred.

Hint: first explain what a declination tide is, then discuss the circumstances under which such a tide is strong and finally, discuss the circumstances at 19 March 2013.

Problem 3

The Sea of Okhotsk (bordered by Russia, China and Japan; see map below) is characterised by strong tidal motion (tidal range of more than 6 m at the northern coast). A major tidal constituent in this area is the K_1 -tide (frequency $\sigma = 7.3 \times 10^{-5} \text{ s}^{-1}$)



$$k = \frac{2\pi}{\lambda}$$

$$\frac{4\pi^2}{\lambda^2} = \frac{4\pi^2}{B^2}$$

$$\lambda = B$$

$$7,3$$

$$14,6$$

$$21,9$$

$$29,2$$

$$36,5$$

$$43,8$$

$$51,1$$

$$58,4$$

$$\frac{2L}{L_0}$$

$$\frac{2}{\sigma}$$

$$\frac{L}{73} =$$

$$f = 5^1 \quad \frac{5^1}{5^1}$$

$$\sqrt{gH} = \sqrt{9,8 \cdot 360} \approx 60 \text{ m/s}$$

$$60 \cdot 60 = 3600$$

$$\sigma^2 = f^2 + \frac{g}{H} \left(k^2 + \frac{n^2 \pi^2}{B^2} \right)$$

For the purposes below the Sea of Okhotsk is schematised as a semi-enclosed rectangular basin with a mean depth of 360 m, a width of 700 km and a length of 1100 km at central latitude $\varphi_0 = 45^\circ$ N. Tides enter the basin at the (partially) open southern boundary. Assume that tidal motion is governed by the frictionless depth-averaged shallow water equations on the f -plane.

- a. Calculate the value of the Rossby radius of deformation of the Sea of Okhotsk, and give a physical interpretation of this length scale.

$$\sigma = \frac{2L}{\sqrt{gH}}$$

- b. Are there Poincaré waves to be expected in the Sea of Okhotsk that have the frequency of the K_1 tide?

If your answer is no, then explain (by means of computations) why not.

If your answer is yes, then indicate in what area you expect them and compute their characteristic length scale.

$$\frac{2,2 \cdot 10^6}{\sqrt{3600}}$$

- c. Assuming the tidal wave to be a Kelvin wave, how many amphidromic points of the K_1 -tide occur in the Sea of Okhotsk?

Explain your answer.

$$\sigma = \frac{2,2}{60} \cdot 10^6$$

$$2,2 \cdot 2,2 = 4,84 \approx 5,6 \approx 6,6$$

$$2,2 \cdot 2,2 =$$

$$\frac{60}{2,2} \cdot 10^{-6} \geq 7,3 \cdot 10^{-5} \quad \frac{L_0}{2L} \geq \sigma, \quad \frac{\sqrt{gH}}{2L} \geq \sigma$$

$$7,3 \cdot 10^{-5}$$

$$\frac{60}{2,2} \cdot 10^6$$