Answers Ocean Waves 03-11-2015

1 a. Critical level z_c defined as

$$U_0(z = z_c) = c$$

where $c \overset{=\sigma/K}{}$ is the phase speed of the waves.

In the present case:

$$c_1 = \left(\frac{g}{K_1} + \frac{T}{\rho} K_1\right)^{1/2} \quad \text{and} \quad c_2 = \left(\frac{g}{K_2} + \frac{T}{\rho} K_2\right)^{1/2}$$

So

$$\frac{u_*}{k} \ln\left(1 + \frac{z_{c,i}}{z_0}\right) = \left(\frac{g}{K_i} + \frac{T}{\rho} K_i\right)^{1/2} \quad \text{where} \quad i = 1, 2$$

$$\Rightarrow \quad \boxed{z_{c,i} = z_0 \left[-1 + \exp\left\{\frac{k}{u_*}\left(\frac{g}{K_i} + \frac{T}{\rho} K_i\right)^{1/2}\right\}\right]}$$

b. Miles shear instability mechanism.

Positive feedback between ocean and atmosphere.

Waves at sea surface create perturbations in flow in atmospheric boundary layer

These perturbations lead to transfer of horizontal momentum in vertical direction.

~~Con~~Divergence of this momentum transport only at critical level $z = z_c$.

So here mean wind is affected: loss of momentum and loss of energy density of atmospheric flow.

This loss of energy is a gain of energy density of the water: wave amplitude increases exponentially

c. The unit of F is m^2/Hz or $m^2 s$: F is the wave variance per frequency.

In other words $\iint F \, d\sigma \, d\theta = \langle \zeta^2 \rangle$ where $\langle \zeta^2 \rangle$ is wave variance.

Unit of σ is s^{-1}, so unit of β is $m^2 s^{-4}$

d. Energy density $\langle E \rangle = \rho g \langle \zeta^2 \rangle$ for gravity waves

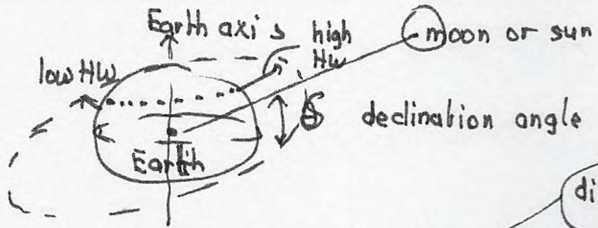
$(\rho g + T K^2)\langle \zeta^2 \rangle$ for capillary gravity waves (but capillary waves have hardly energy at this stage)

$\Rightarrow$ Energy density of high frequency waves

$$\langle E \rangle_{high} = \rho g \int_{\sigma_p}^{\infty} \int_{\theta_w - \frac{\pi}{2}}^{\theta_w + \frac{\pi}{2}} \beta \sigma^{-5} \cos^2(\theta - \theta_w) \, d\theta \, d\sigma = \rho g \, \frac{1}{2}\pi \, \frac{1}{4} \sigma_p^{-4}$$

2a. Dominant period : 1 HW, 1LW per day, so period ~ 24 h : diurnal tide.
Diurnal tides result from declination : line connecting centre of mass Earth and that of ~~a~~ celestial body is not in equatorial plane.

So



difference local gravitational force and gravitational force in centre of mass Earth

The equilibrium tide (resulting from tidal force balancing pressure gradient force is indicated by the dashed curve.
Thus an observer at latitude φ on rotating Earth will experience in 1 cycle of ~24 h a high HW and low HW. Thus semi-diurnal + diurnal tide.
The smaller δ, the weaker the diurnal tide.

b. Pure coincidence, the magnitude of the diurnal tide is not determined by full moon, last quarter etc, but by the declination angle δ. Apparently (and indeed) on 271015 the declination angle of the moon was small. So was that of the sun (the latter was 0 on 210915 and will become 23.5° on 211215)
During full moon : period ~ $12^{h}\,25^{m}$, the semi-diurnal tides are close to maximum at that time.

c. Yes, in particular because the distance moon - Earth will be different
That distance varies on a period of 27.5 day and it causes tidal forces to be stronger when the distance is small, and weaker when the distance is large.
From moon data : distance larger in that period → tidal range smaller

3a. Two types of waves:

Kelvin waves: $\sigma^2 = gHK^2$ so $\lambda = \frac{2\pi}{K} = \sqrt{gH}\,T$

where $T = 2\pi/\sigma$ the period

Poincaré waves: $\sigma^2 = f^2 + gH\left(k^2 + \frac{n^2\pi^2}{B^2}\right)$

so $\lambda = \frac{2\pi}{K}$ and $K_n = \left(\frac{\sigma^2 - f^2}{gH} - \frac{n^2\pi^2}{B^2}\right)^{1/2}$

$T = 44700$ s, $\sigma = 1.4\cdot 10^{-4}\ s^{-1}$

So $\boxed{\lambda_{Kelvin} \simeq 7070 \text{ km}}$

Furthermore $K_1 = (3.84\cdot 10^{-13} - 4\cdot 10^{-13})^{1/2}\ m^{-1}$ ↯ no solution

So no travelling Poincaré waves

However, when $\begin{cases} g = 9.81\ ms^{-2} \\ \sigma = 1.405\cdot 10^{-4}\ s^{-1} \end{cases}$ is used: n=1 Poincaré wave is present

(with an extremely large wavelength)

b. • Kelvin waves, trapped on east/west coast, length scale $R = \frac{\sqrt{gH}}{f}$

Rossby radius of deformation

~ 1600 km

• Poincare waves, trapped on north coast

length scale $\left[\ell_n = s_n^{-1}\right.$ from dispersion relation

$$\sigma^2 = f^2 + gH\left(-s_n^2 + \frac{n^2\pi^2}{B^2}\right) \rightarrow S_n = \left(\frac{f^2 - \sigma^2}{gH} + \frac{n^2\pi^2}{B^2}\right)^{1/2}$$

Note $s_1 < s_2 < \cdots$

Here $s_1 \simeq (1.6\cdot 10^{-14})^{1/2}\ m^{-1} \simeq 1.26\cdot 10^{-7}\ m^{-1}$ if $\sigma = 1.4\cdot 10^{-4} s^{-1}$ is used

So $\ell_1 \simeq 8000$ km (so they occur in quite a large part of the domain)

If $\begin{cases} g = 9.81\ ms^{-2} \\ \sigma = 1.41\cdot 10^{-4}\ s^{-1} \end{cases}$ is used: n=1 is not trapped

$\Rightarrow$ compute $s_2 \simeq 1.1\cdot 10^{-6}\ m^{-1}$ $\rightarrow \ell_2 = s_2^{-1} \simeq 900$ km