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In 2008/2009, the course NS-TP401M was given by B.de Wit.

Quantum Field Theory (NS-TP401M)
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Question 1. Spinor fields (6.5 points)

Consider a theory of N spinor field ψ_i, $i = 1, \ldots, N$, on two-dimensional Minkowski space, with Lagrangian density

$$\mathcal{L} = \bar{\psi}_i i \partial\!\!\!/ \psi_i + \frac{g^2}{2}(\bar{\psi}_i \psi_i)^2, \tag{1}$$

where a sum over i is understood. An explicit form of the two-dimensional γ-matrices is given by

$$\gamma^0 \equiv \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \gamma^1 \equiv \sigma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \tag{2}$$

with σ^i denoting the Pauli matrices. We also have $\gamma^5 := \gamma^0\gamma^1$.

a) Verify that the γ-matrices satisfy the Dirac (Clifford) algebra.

b) Show that $\mathcal{L}$ is invariant under the (discrete) chiral symmetry $\psi_i \to \gamma^5\psi_i$, $\forall i$, and that this invariance is broken by adding a fermionic mass term $m\bar{\psi}_i\psi_i$ to $\mathcal{L}$. Which other symmetries does (1) possess? (Explain!)

c) Recalling the definition $S^{\mu\nu} := \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ for the generators of the spinor representation of the Lorentz algebra, compute the corresponding finite group action of the Lorentz group on the spinors ψ. (Since we are in two dimensions, this is the group $SO(1,1)$). Show how γ^5 can be used to construct projectors on spinor subspaces which transform separately under $SO(1,1)$.

d) Determine the mass dimension of the spinor fields and the coupling constant g. Thus, is the theory renormalizable (superficially, according to power-counting)?

Question 2. One-loop diagrams (8.5 points)

Consider a theory (in four-dimensional Minkowski space) with massive Dirac fermions ψ and real massive scalar particles ϕ, with an interaction term of the form $\mathcal{L}_{int} = g\bar{\psi}\phi\psi$.

a) Write down the action of the theory and draw the Feynmann diagrams which correspond to the lowest-order (in the coupling g) corrections to (i) the fermion propagator, (ii) the scalar field propagator and (iii) the interaction vertex. (These are the connected one-loop diagrams.)

b) For the one particle irreducible diagrams from part (a) - those that cannot be split into two by removing a single line - write down the associated truncated amplitudes (i.e. omitting the propagators of the external legs).

c) Regularizing any infinities by introducing a Lorentz-invariant momentum cut-off Λ, compute the leading and subleading terms in Λ contributing at one-loop order to the truncated amplitude of (ii) by performing all integrations explicitly. (Do all calculations "exactly", allowing for finite variable shifts in the momentum integrals, and then introduce Λ.)

[Hint: The identity

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{(xA + (1-x)B)^2}, \tag{3}$$

may come in handy.]

Question 3. Computing a propagator (5 points)

When working with QED it is sometimes convenient to give the photon a (small) mass m at some intermediate stage of the calculation, corresponding to using the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu, \; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{4}$$

for the electromagnetic field. By Fourier transformation, determine the propagator in momentum space for the massive photon from (4).