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In 2009/2010, the course NS-TP401M was given by R. Loll.

Quantum Field Theory, Retake Exam (NS-TP401M) 21-04

You must obtain at least 10 of the 20 points to pass the exam. The use of auxilary materials such as books, notes, calculators, laptops etc. is not permitted. Please hand in all sheets you used for calculations.

Question 1 - Computing loop corrections (8.5 points)

Consider a theory (in four dimensional Minkowski space) with massive Dirac fermions ψ and real massive scalar particles ϕ, with an interaction term of the form $\mathscr{L}_{int} = g\overline{\psi}\phi\psi$.

a) Write down the action of the theory and draw the Feynman diagrams which corrspond to the lowest-order (in the coupling g) corrections to (i) the fermion propagator, (ii) the scalar field propagator and (iii) the interaction vertex. (These are the connected one-loop diagrams).

b) For the diagrams from part (a) that cannot be split into two disconnected parts by removing a single line (they are called ¨one-particle irreducible diagrams¨, write down the associated truncated amplitudes (i.e. omitting the propagators of the external legs).

c) Regularizing any infinities by introducing a Lorentz-invariant momentum cut-off Λ, compute the leading and subleading terms in Λ^2 contributing at one-loop order to the truncated amplitudes of (ii) (i.e., the scalar field propagator) by performing all integrations explicity. (First, reduce the amplitude to elementary momentum integrals, possibly allowing for finite variable shifts in the momenta. Evaluate these further, regulating any divergences that may appear in terms of Λ.)

[Hint: The identity

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{(xA + (1-x)B)^2} \tag{1}$$

may come in handy.]

Question 2 - Field theory with fermions (11.5 points)

Consider the classical field theory based on the action

$$S[\psi_1, \psi_2, A] = \int d^4x \left(\overline{\psi_1} \imath \not{D} \psi_1 + \overline{\psi_2}(\imath \not{D} - m)\psi_2 - \tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} \right) \tag{2}$$

where ψ_1 and ψ_2 denote a massless and a massive fermion field, $D_\mu = \partial_\mu - \imath e A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

a) Determine the classical equations of motion for all of the fields.

b) How many global invariances does the action (2) have, i.e. how many one-parameter groups of global transformations leave S invariant? For those global transformations which affect the fermion fields only, prove the invariance of S explicity and then compute the associated conserved current(s) according to Noether´s theorem and verify by use of the equations of motion that they are indeed conserved.

c) Compute the conserved current associated with time translations, What is the conserved charge (calculate it), and what is its role?

d) Drop the itneraction term from S and compute the free-field momentum space propagator of the vector field A. In case you encounter any difficulties, how are they resolved? Present an explicit calculation.